Assignment

In this assignment, all the questions are intended to be completed algebraically.

1. At a local golf course, on the par 3 hole, Linda used a seven iron to reach the green. Her golf ball followed the path of a parabola, approximated by the function

$$h(t) = -5t^2 + 25t + 0.05$$

where t is the number of seconds which have elapsed since Linda hit the ball,
and $h(t)$ is the height, in metres, of the ball above the ground after t seconds.

a) Write the function in standard form.

$h(t) = -5(t^2 - 5t\ \ ) + 0.05$
$= -5(t^2 - 5t + 6.25 - 6.25) + 0.05$
$= -5(t - 2.5)^2 + 31.25 + 0.05$

$h(t) = -5(t-2.5)^2 + 31.3$

b) Find the height of the golf ball 2 seconds after the ball is hit.

$h(2) = -5(2-2.5)^2 + 31.3 = 30.05$ height = 30.05 m

c) Find the maximum height reached by the golf ball. 31.3 m

d) How many seconds did it take for the golf ball to reach its maximum height? 2.5 seconds

e) How high, in centimetres, did Linda tee up her golf ball before she hit it?

$h(0) = 0.05$ $0.05\text{m} = 5\text{cm}$ 5 cm high

f) How long, to the nearest tenth of a second, did it take for the golf ball to hit the ground?

$h(t) = 0$ $-5t^2 + 25t + 0.05 = 0$

$a = -5$
$b = 25$
$c = 0.05$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \dfrac{-25 \pm \sqrt{(25)^2 - 4(-5)(0.05)}}{2(-5)}$

$t = \dfrac{-25 \pm \sqrt{626}}{-10}$

$t = -0.00199\ldots$ or $5.00199\ldots$

(reject negative root)

time = 5.0 seconds

2. The sum of a number, x, and its reciprocal is $\frac{29}{10}$.
Form an equation and find the original number.

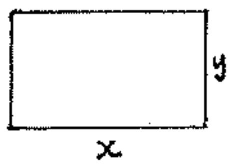
$x + \dfrac{1}{x} = \dfrac{29}{10}$ $(\times 10x)$

$10x^2 + 10 = 29x$
$10x^2 - 29x + 10 = 0$
$10x^2 - 25x - 4x + 10 = 0$
$5x(2x-5) - 2(2x-5) = 0$

$(2x-5)(5x-2) = 0$
$x = \frac{5}{2}$ or $\frac{2}{5}$

The original number is $\frac{5}{2}$ or $\frac{2}{5}$.

3. The perimeter of a rectangular plot of land is 84 metres and its area is 320 metres^2. If the length of the plot is represented by x metres, form a quadratic equation in x, and solve it to find the length and width of the plot.



$2x + 2y = 84$

$x + y = 42$

$y = 42 - x$

length = x metres

width = $42 - x$ metres

area = $x(42 - x)$ m^2

$x(42 - x) = 320$

$42x - x^2 = 320$

$0 = x^2 - 42x + 320$

$(x - 10)(x - 32) = 0$

$x = 10, 32$

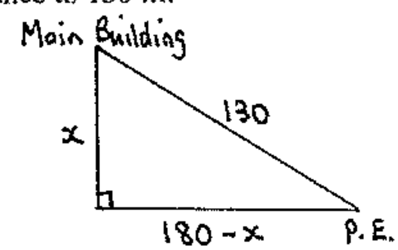
if $x = 10$, $y = 32$

if $x = 32$, $y = 10$

length and width are $\underline{\underline{32\text{m and } 10\text{m}}}$.

4. The paved walkway from the main school building to the Physical Education block at a school is "L" shaped, with the total distance being 180 metres. A student, taking a short cut diagonally across the grass, shortens the distance to 130 m.

a) Draw a sketch to illustrate this information.



b) If one of the "L" shaped sides has a length of x metres, state the length of the other "L" shaped side in terms of x.

$180 - x$ metres

c) Use the Pythagorean Theorem to write a quadratic equation in x. Solve the equation to determine the length of the two legs of the paved walkway. Answer to the nearest tenth of a metre.

$x^2 + (180 - x)^2 = 130^2$

$x^2 + 32400 - 360x + x^2 = 16900$

$2x^2 - 360x + 15500 = 0$

$x^2 - 180x + 7750 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1 \quad b = -180 \quad c = 7750$

$$x = \frac{180 \pm \sqrt{(-180)^2 - 4(1)(7750)}}{2(1)}$$

$$x = \frac{180 \pm \sqrt{1400}}{2}$$

$x = 71.3$ or 108.7

$180 - x = 108.7$ or 71.3

The two legs are $\underline{\underline{71.3\text{ m and } 108.7\text{ m}}}$

5. A stone is thrown vertically upward at a speed of 22 m/s. Its height, h metres, after t seconds, is given approximately by the function $h(t) = 22t - 5t^2$.
Use this formula to find, to the nearest tenth of a second, when the stone is 15 metres up and explain the double answer.

$h(t) = 22t - 5t^2$

$15 = 22t - 5t^2$

$5t^2 - 22t + 15 = 0$

$t = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = 5 \quad b = -22 \quad c = 15$

$t = \dfrac{22 \pm \sqrt{(-22)^2 - 4(5)(15)}}{2(5)}$

$t = \dfrac{22 \pm \sqrt{184}}{10}$ $\qquad t = 0.8, 3.6$

The stone is 15 m up after 0.8 sec. and 3.6 sec.

There are two answers as the stone goes up and then comes down.

Multiple Choice **6.** Two numbers have a difference of 20. When the squares of the numbers are added together, the result is a minimum. The larger of the two numbers is

A. 0
B. 10
C. 20
D. 30

Let the numbers be x and $x + 20$

$f(x) = x^2 + (x+20)^2$ is to be a minimum

$= x^2 + x^2 + 40x + 400$

$= 2x^2 + 40x + 400$

$= 2(x^2 + 20x + 100 - 100) + 400$

$= 2(x+10)^2 - 200 + 400$

$= 2(x+10)^2 + 200$

minimum value when $x = -1$

$x + 20 = -10 + 20 = 10$

larger number is 10.

Numerical Response **7.** A springboard diver's height, in metres, above the water, is given by the formula

$$h(t) = -5t^2 + 8t + 4$$

where t is the number of seconds which have elapsed since the start of the dive,
and $h(t)$ is the height, in metres of the diver above the water after t seconds.

The time taken, to the nearest tenth of a second, for the diver to enter the water is _____.

(Record your answer in the numerical response box from left to right.)

2	.	0	

$-5t^2 + 8t + 4 = 0$

$t = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = -5$, $b = 8$, $c = 4$

$t = \dfrac{-8 \pm \sqrt{(8)^2 - 4(-5)(4)}}{2(-5)}$

$t = \dfrac{-8 \pm \sqrt{144}}{-10}$

$t = -0.4, 2$

reject -0.4 since $t > 0$

$t = 2.0$

8. One positive integer is 3 greater than 4 times another positive integer.
If the product of the two integers is 76, then the sum of the two integers is ______ .

(Record your answer in the numerical response box from left to right.)

2	3		

Let the integers be x and $4x+3$

$x(4x+3) = 76$

$4x^2+3x-76=0$

$4x^2+19x-16x-76=0$

$x(4x+19)-4(4x+19)=0$

$(4x+19)(x-4)=0$

$x = \frac{-19}{4}$ or 4

reject $\frac{-19}{4}$ (not an integer)

$x=4$

$4x+3 = 4(4)+3 = 19$

the integers are 4 and 19

sum = 23

9. A whole number is multiplied by 5 and added to 3 times its reciprocal to give a sum of 16.
The number is ______ .

(Record your answer in the numerical response box from left to right.)

3			

Let the whole number be x.

$5x + \frac{3}{x} = 16 \quad (\times x)$

$5x^2+3=16x$

$5x^2-16x+3=0$

$5x^2-x-15x+3=0$

$x(5x-1)-3(5x-1)=0$

$(5x-1)(x-3)=0$

$x = \frac{1}{5}$ or 3

reject $\frac{1}{5}$ (not a whole number)

$x=3$

10. Use the vertex formula to determine the coordinates of the vertex of the graph of each of the following functions. State the maximum or minimum value of each function.

a) $f(x) = 5x^2+3x-2$

$a=5 \quad b=3 \quad c=-2$

$\frac{-b}{2a} = \frac{-3}{2(5)} = -\frac{3}{10}$

$\frac{4ac-b^2}{4a} = \frac{4(5)(-2)-(3)^2}{4(5)} = \frac{-49}{20}$

vertex $\left(-\frac{3}{10}, -\frac{49}{20}\right)$

min. value $= \frac{-49}{20}$

b) $f(x) = -3x^2-7x-1$

$a=-3 \quad b=-7 \quad c=-1$

$\frac{-b}{2a} = \frac{7}{2(-3)} = -\frac{7}{6}$

$\frac{4ac-b^2}{4a} = \frac{4(-3)(-1)-(-7)^2}{4(-3)} = \frac{37}{12}$

vertex $\left(-\frac{7}{6}, \frac{37}{12}\right)$

max. value $= \frac{37}{12}$

c) $f(x) = x^2+9x+4$

$a=1 \quad b=9 \quad c=4$

$\frac{-b}{2a} = \frac{-9}{2(1)} = -\frac{9}{2}$

$\frac{4ac-b^2}{4a} = \frac{4(1)(4)-(9)^2}{4(1)} = -\frac{65}{4}$

vertex $\left(-\frac{9}{2}, -\frac{65}{4}\right)$

min. value $= \frac{-65}{4}$