L1-4 Practice Test

key

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Check Your Understanding

Practise

1. State the coordinates of the vertex and the number of x-intercepts for each of the following functions.

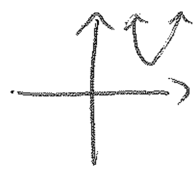
a) $y = (x - 3)^2 + 5$

$p =$ 3 $q =$ 5

vertex: (3 , 5)

a > 0; the graph opens up
(< or >) (*upward* or *downward*)

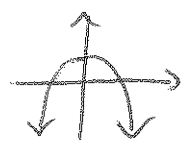
q > 0; there are no x-intercepts.
(< or >)



b) $y = -4x^2 + 1$

vertex $(0, 1)$

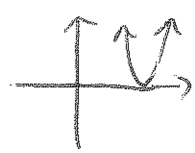
graph opens down, $q > 0$ → two x-intercepts



c) $y = \frac{2}{3}(x - 11)^2$

vertex $(11, 0)$

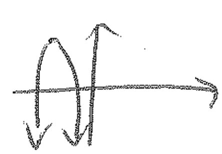
graph opens up. → one x-intercept



d) $y = -\left(x + \frac{1}{2}\right)^2 + \frac{7}{3}$

vertex $(-1/2, 7/3)$

graph opens down → two x-intercepts



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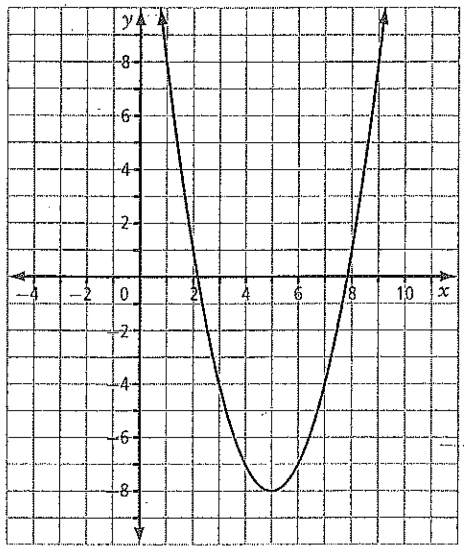
2. State the direction of opening, the equation of the axis of symmetry, and the maximum or minimum value for each of the following.

a)



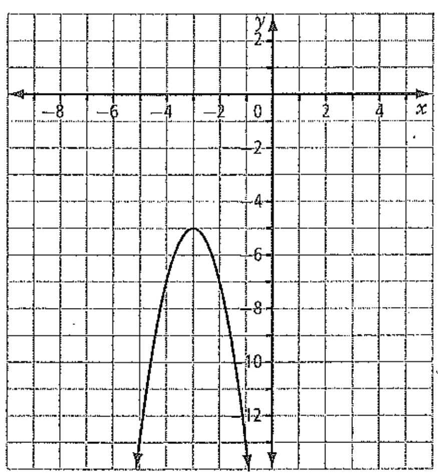
$p =$ 5 $q =$ −8

The graph opens up.
(*upward* or *downward*)

The equation of the axis of symmetry is $x = 5$.

The minimum value is −8.
(*maximum* or *minimum*)

b)



$p =$ −3 $q =$ −5

The graph opens down.
(*upward* or *downward*)

The equation of the axis of symmetry is $x = -3$.

The maximum value is −5.
(*maximum* or *minimum*)

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3. Describe how to obtain the graph of each function from the graph of $y = x^2$. State the domain and the range for each. Then, graph the function.

a) $y = (x + 4)^2 - 2$

$a =$ 1 $p =$ -4 $q =$ -2

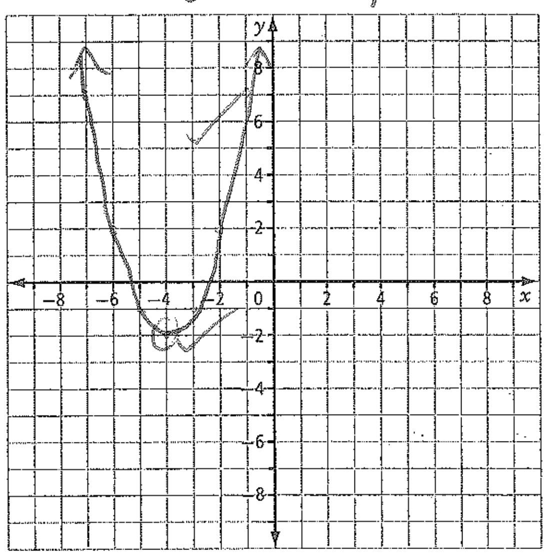
The graph opens up.
(*upward* or *downward*)

The minimum value is -2.
(*maximum* or *minimum*)

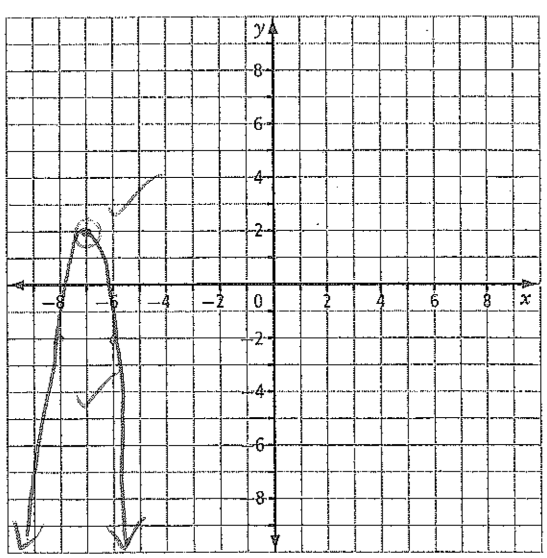
The graph of $y = x^2$ is translated 4 units to the left and
(*left* or *right*)

2 units down.
(*up* or *down*)

The domain is $\{x \mid x \in \mathbb{R}\}$. The range is $\{y \mid y \geq -2, y \in \mathbb{R}\}$



b) $y = -4(x + 7)^2 + 2$



$a = -4$, $p = -7$, $q = 2$

opens down

max value = 2

7 units left, 2 units up

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \leq 2, y \in \mathbb{R}\}$

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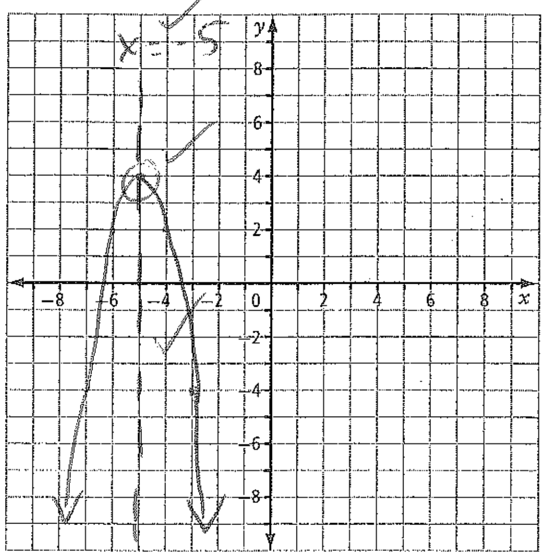
4. Sketch each of the following. Label the vertex and axis of symmetry. State the domain and range.

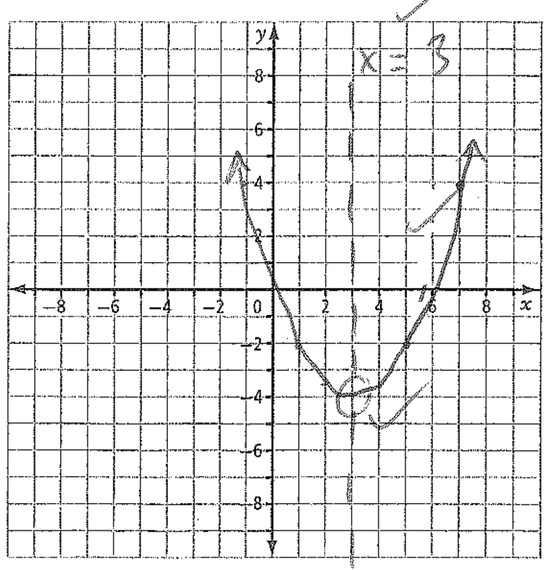
a) $y = -2(x + 5)^2 + 4$



D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \leq 4, y \in \mathbb{R}\}$

b) $y = \frac{1}{2}(x - 3)^2 - 4$



D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq -4, y \in \mathbb{R}\}$

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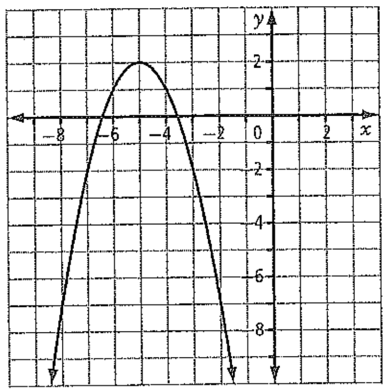
5. Determine the quadratic function in vertex form for each parabola.

a)



$p =$ -5

$q =$ 2

$a =$ -1

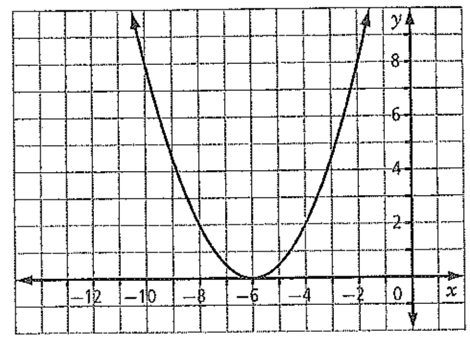
use (-4,1) to find a

$1 = a(-4+5)^2 + 2$

$-1 = a$

function: $y = -(x+5)^2 + 2$

b)



$p =$ -6

$q =$ 0

$a =$ 1/2

use (-4,2) to find a

$2 = a(-4+6)^2 + 0$

$1/2 = a$

function: $y = 1/2(x+6)^2$

6. Write the quadratic function in vertex form that has the given characteristics.

a) vertex at (0, 4), congruent to $y = 5x^2$

$a =$ 5 $p =$ 0 $q =$ 4

function: $y = 5x^2 + 4$

b) vertex at (3, 0), passing through (4, −2)

Substitute the coordinates of the vertex and the point (4, −2) into $y = a(x - p)^2 + q$ to determine a.

$y = a(x-3)^2 + 0$

$-2 = a(4-3)^2$

$-2 = a$

$\boxed{y = -2(x-3)^2}$

c) vertex (1, −1), with y-intercept 3

Substitute the coordinates of the vertex and the coordinates of the y-intercept into $y = a(x - p)^2 + q$ to determine a.

$y = a(x-1)^2 - 1$

$3 = a(0-1)^2 - 1$

$4 = a$

$\boxed{y = 4(x-1)^2 - 1}$

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Apply

7. Determine the corresponding point on the transformed graph for the point (−1, 1) on the graph of $y = x^2$.

a) $y = x^2$ is transformed to $y = (x + 5)^2 - 1$.

For $y = (x + 5)^2 - 1$, $p =$ __-5__ and $q =$ __-1__.

Apply the horizontal translation of 5 units to the left and the vertical translation of 1 unit down to the point (−1, 1): (−1 + __-5__, 1 + __-1__)

The corresponding point of (−1, 1) after the graph is transformed is (__-6__, __0__).

This question should help you complete #10 on page 158 of *Pre-Calculus 11*.

b) $y = x^2$ is transformed to $y = 2(x - 2)^2 - 3$.

For $y = 2(x - 2)^2 - 3$, $a =$ __2__, $p =$ __2__, and $q =$ __-3__.

Apply the multiplication of the y-values by a factor of 2 to the point (−1, 1):

(−1, __2__ (1))

Then, apply the horizontal translation of 2 units to the right and the vertical translation of 3 units down to the point (−1, 2): (−1 + __2__, 2 + __-3__)

The corresponding point of (−1, 1) after the graph is transformed is (__1__, __-1__).

c) $y = x^2$ is transformed to $y = \frac{1}{2}(x + 1)^2 + 4$.

$(-1, 1) \rightarrow (-1, \frac{1}{2}) \rightarrow (-2, \frac{1}{2}) \rightarrow (-2, \frac{9}{2})$

vertical stretch

horiz translation

vertical translation

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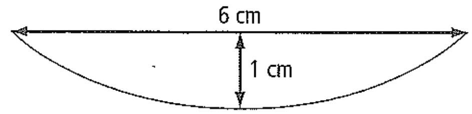
8. Parabolic mirrors are often used in lights because they give a focused beam. Suppose a parabolic mirror is 6 cm wide and 1 cm deep, as shown.



a) Suppose the vertex of the mirror is at the origin. Determine the quadratic function in vertex form that describes the shape of the mirror.

The coordinates of the vertex are $(0, 0)$.

The coordinates of one endpoint of the mirror are $(3, 1)$.

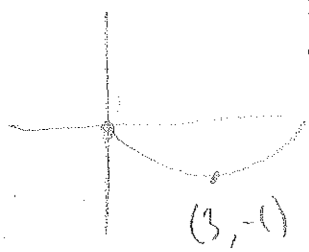
Use the coordinates of the vertex and the endpoint to determine *a*.

$$y = ax^2$$
$$1 = a(3)^2$$
$$1/9 = a$$

If the vertex is at the origin, the function is $y = \frac{1}{9}x^2$.

b) Now suppose the origin is at the left outer edge of the mirror. Determine the quadratic function in vertex form that describes the mirror.

The coordinates of the vertex are $(3, -1)$



$$y = a(x-3)^2 - 1$$
$$0 = a(0-3)^2 - 1$$
$$1/9 = a$$

If the origin is at the left endpoint, the function is $y = \frac{1}{9}(x-3)^2 - 1$

c) Compare your functions in parts a) and b). How are they similar? How are they different?

a parameter is the same

p, q. values are different

Domain is the same

Range is different

Axis of symmetry is different

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