Exponents and Radicals Lesson #2:
Introduction to Radicals

Review

Recall the following notes from Lesson #1.

Square Roots : All positive numbers have two square roots: one a positive number and the other a negative number. The positive square root is called the **principal square root** and is denoted by the symbol $\sqrt{\ }$.

The square roots of a perfect square are rational numbers.
e.g. the square roots of 16 are 4 and –4. **NOTE:** $\sqrt{16} = 4$ <u>only</u>.

The square roots of a non-perfect square are irrational numbers.
e.g. the square roots of 17 are $\sqrt{17}$ and $-\sqrt{17}$.

Cube Roots: All numbers (positive and negative) have one cube root, denoted by the symbol $\sqrt[3]{\ }$.

- The cube root of a perfect cube is a rational number.
  e.g. the cube root of 1 000 is 10, i.e. $\sqrt[3]{1\,000} = 10$.
  the cube root of –27 is –3, i.e. $\sqrt[3]{-27} = -3$.
- The cube root of a non-perfect cube is an irrational number.
  e.g. the cube root of 49 is $\sqrt[3]{49}$, which is irrational.

Other Roots

Complete the following statements.

$4 \times 4 = 16$ so a square root of 16 is 4 or $\sqrt{16} =$ 4 .
$5 \times 5 \times 5 = 125$ so the cube root of 125 is 5 or $\sqrt[3]{125} =$ 5 .
$2 \times 2 \times 2 \times 2 = 16$ so a fourth root of 16 is 2 or $\sqrt[4]{16} =$ 2 .
$10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1\,000\,000$ so sixth root of 1000000 is 10
or $\sqrt[6]{1\,000\,000} = 10$

Class Ex. #1

Mentally evaluate, where possible, using the real number system.

a) $\sqrt{49}$ $= 7$ **b)** $\sqrt[3]{-64}$ $= -4$ **c)** $\sqrt[4]{10\,000}$ $= 10$

d) $\sqrt[5]{\dfrac{1}{32}} = \dfrac{\sqrt[5]{1}}{\sqrt[5]{32}} = \dfrac{1}{2}$ **e)** $\sqrt[4]{-16}$ not possible **f)** $2\sqrt[3]{125}$ $= 2(5) = 10$

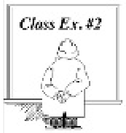
Complete Assignment Question #1 - #2

Using the $\sqrt[x]{\ }$ Feature of a Calculator

Use the following procedure to determine $\sqrt[4]{10\,000}$ on a calculator.

1. Press $\boxed{4}$. **2.** Press $\boxed{\text{MATH}}$. **3.** Choose **5:** $\sqrt[x]{\ }$.

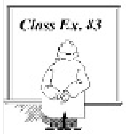
4. Press 10 000 . **5.** Press $\boxed{\text{ENTER}}$. The answer will be 10.



Use a calculator to evaluate.

a) $\sqrt[5]{1\,024}$ **b)** $\sqrt[7]{-2\,187}$ **c)** $-3\sqrt[4]{50\,625}$ **d)** $\sqrt[3]{\frac{216}{125}}$ $= \frac{\sqrt[3]{216}}{\sqrt[3]{125}}$

$= 4$ $= -3$ $= -45$ $= 6/5$



Evaluate to the nearest hundredth.

a) $\sqrt[5]{125}$ **b)** $\sqrt[6]{0.5}$ **c)** $\frac{2}{3}\sqrt[4]{1\,000}$

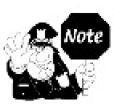
$= 2.6265....$ $= 0.8908....$ $= 3.7489....$

$= 2.63$ $= 0.89$ $= 3.75$

Radicals

Numbers like $\sqrt{30}$, $\sqrt[3]{125}$, $\sqrt[4]{15}$, $\sqrt[6]{1\,000\,000}$ etc. are examples of **radicals**.

In fact, any expression of the form $\boxed{\sqrt[n]{x}}$, where $n \in N$, is called a radical.
n is called the **index**. In a number like $\sqrt{30}$ the index is 2.
x is called the **radicand** and $\sqrt{\ }$ is called the **radical sign**.
If the index in a radical is even, then the radicand must be positive.



- When the index is not written in the radical, as in square root, it is assumed to be 2.
- The index is the number of times the radical must be multiplied by itself to equal the radicand.

Class Ex. #4

Identify the index and the radicand in each of the following.

a) $\sqrt[5]{75}$ **b)** $\sqrt{50}$ **c)** $5\sqrt[3]{-\frac{1}{10}}$

index: 5 index: 2 index: 3

radicand: 75 radicand: 50 radicand: $-\frac{1}{10}$

Review

Recall the following results from Lesson #1, Assignment Question #10.

$\sqrt{9} \times \sqrt{4}$ is equal to $\sqrt{9 \times 4}$ $\sqrt{9} + \sqrt{4}$ is **not** equal to $\sqrt{9+4}$

$\sqrt{9} \div \sqrt{4}$ is equal to $\sqrt{9 \div 4}$ $\sqrt{9} - \sqrt{4}$ is **not** equal to $\sqrt{9-4}$

The calculations above are examples of some general rules involving radicals.

Note

i) The product(quotient) of the roots of two numbers is equal to the root of the product(quotient) of the two numbers.

ii) The sum (difference) of the roots of two numbers is NOT equal to the root of the sum (difference) of the two numbers.

In general $\boxed{\sqrt{a} \times \sqrt{b} = \sqrt{ab}}$ where $a, b \geq 0$ and $\boxed{\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}}$ where $a \geq 0,\ b > 0$.

Class Ex. #5

State whether each statement is true or false.

a) $\sqrt{3} \times \sqrt{6} = \sqrt{18}$ **b)** $\frac{\sqrt{20}}{\sqrt{10}} = \sqrt{2}$ **c)** $\sqrt{16+9} = \sqrt{16} + \sqrt{9}$

✓ ✓ ✗

Class Ex. #6

Write the following as a single radical in the form $\sqrt{x}$.

a) $\sqrt{8} \times \sqrt{3}$ **b)** $\sqrt{7} \times 3$ **c)** $\frac{\sqrt{50}}{\sqrt{10}}$ **d)** $\frac{\sqrt{\sqrt{100}}}{\sqrt{2}}$ $= \frac{\sqrt{10}}{\sqrt{2}}$

$= \sqrt{8 \times 3}$ $= \sqrt{21}$ $= \sqrt{\frac{50}{10}} = \sqrt{5}$ $= \sqrt{\frac{10}{2}} = \sqrt{5}$

$= \sqrt{24}$

Class Ex. #7

Express as a product of radicals.

a) $\sqrt{77}$ **b)** $\sqrt{27}$ **c)** $\sqrt{40}$

$= \sqrt{11} \times \sqrt{7}$ $= \sqrt{9} \times \sqrt{3}$ $= \sqrt{8} \times \sqrt{5}$

(and others)

Complete Assignment Questions #3 - #13

Do #1-13