Exponents and Radicals Lesson #6:
Rational Exponents - Part One

Review of the Exponent Laws

The exponent laws with integral exponents and numerical and variable bases were covered in previous math courses.

Complete the table as a review of the exponent laws.

Numerical Bases	Variable Bases	Exponent Laws
$8^3 \times 8^2 = (8 \cdot 8 \cdot 8)(8 \cdot 8)$ $= 8^5$ or 8^{3+2}	$a^3 \times a^2 = (a \cdot a \cdot a)(a \cdot a)$ $= a^5$ or a^{3+2}	**Product Law** $(a^m)(a^n) = a^{m+n}$
$8^3 \div 8^2 = \dfrac{8 \cdot 8 \cdot 8}{8 \cdot 8}$ $= 8^1$ or $8^{\ -\ }$	$a^3 \div a^2 = \dfrac{a \cdot a \cdot a}{a \cdot a}$ $= a$ or a^{3-2}	**Quotient Law** $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$
$(8 \cdot 7)^3 = (8 \cdot 7)(8 \cdot 7)(8 \cdot 7)$ $= (8 \cdot 8 \cdot 8)(\qquad)$ $= 8^3 \cdot 7^3$	$(a \cdot b)^3 = (a \cdot b)(a \cdot b)(a \cdot b)$ $= (a \cdot a \cdot a)(b \cdot b \cdot b$ $= a^3 b^3$	**Power of a Product Law** $(ab)^m = a^m b^m$
$\left(\dfrac{8}{7}\right)^3 = \left(-\right)\left(-\right)\left(-\right)$ $= \dfrac{8^3}{7^3}$	$\left(\dfrac{a}{b}\right)^3 = \left(\dfrac{a}{b}\right)\left(\dfrac{a}{b}\right)\left(\dfrac{a}{b}\right)$ $= \dfrac{a^3}{b^3}$	**Power of a Quotient Law** $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ $(b \neq 0)$
$(8^3)^2 = (8^3)(8^3)$ $= (\qquad)(\qquad)$ $= 8^6$ or $8^{\ \times\ }$	$(a^3)^2 = (a^3)(a^3)$ $= (\qquad)(\qquad)$ $= a^6$ or $a^{3 \times 2}$	**Power of a Power Law** $(a^m)^n = a^{m \times n}$

Negative Exponent Law

$$a^{-n} = \frac{1}{a^n} \quad \text{or} \quad \frac{1}{a^{-n}} = a^n$$

Investigating the Meaning of $a^{\frac{1}{n}}$

a) Complete and evaluate the following.

i) $\sqrt{5} \cdot \sqrt{5} = \sqrt{25} = 5$ ii) $5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 5^{\frac{1}{2}+\frac{1}{2}} = 5^{1} = 5$

Deduce a meaning for $5^{\frac{1}{2}}$ in radical form. $\sqrt{5}$

b) Complete and evaluate the following.

i) $\sqrt[3]{2} \cdot \sqrt[3]{2} \cdot \sqrt[3]{2} = \sqrt[3]{8} = 2$ ii) $2^{\frac{1}{3}} \cdot 2^{\frac{1}{3}} \cdot 2^{\frac{1}{3}} = 2^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 2^{1}$

Deduce a meaning for $2^{\frac{1}{3}}$. $\sqrt[3]{2}$

c) Write the following in radical form and evaluate manually. Verify with a calculator.

i) $25^{\frac{1}{2}} =$ **ii)** $64^{\frac{1}{3}} =$ **iii)** $81^{\frac{1}{4}} =$

d) Write the following in radical form.

i) $x^{\frac{1}{2}} =$ **ii)** $b^{\frac{1}{3}} =$ **iii)** $p^{\frac{1}{10}} =$ **iv)** $\boxed{a^{\frac{1}{n}} = \sqrt[n]{a}}$

Investigating the Meaning of $a^{\frac{m}{n}}$

1. a) Complete and evaluate the following.

i) $\sqrt{5^3} \cdot \sqrt{5^3} = 5^3 = 125$ ii) $5^{\frac{3}{2}} \cdot 5^{\frac{3}{2}} = 5^{\frac{3}{2}+\frac{3}{2}} = 5^{\frac{6}{2}} = 5^3 = 125$

Deduce a meaning for $5^{\frac{3}{2}}$ in radical form. $\sqrt{5^3}$ or $(\sqrt{5})^3$

b) Complete and evaluate the following.

i) $\sqrt[3]{2^2} \cdot \sqrt[3]{2^2} \cdot \sqrt[3]{2^2} = 2^2 = 4$ ii) $2^{\frac{2}{3}} \cdot 2^{\frac{2}{3}} \cdot 2^{\frac{2}{3}} = 2^{\frac{2}{3}+\frac{2}{3}+\frac{2}{3}} = 2^{\frac{6}{3}} = 2^2 = 4$

Deduce a meaning for $2^{\frac{2}{3}}$. $\sqrt[3]{2^2}$ or $(\sqrt[3]{2})^2$

c) Write the following in radical form.

i) $x^{\frac{5}{3}} =$ **ii)** $b^{\frac{4}{5}} =$ **iii)** $p^{\frac{5}{2}} =$ **iv)** $\boxed{a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ or } (\sqrt[n]{a})^m}$

2. a) Evaluate i) $8^{\frac{2}{3}} = (8^{\frac{1}{3}})^2 = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$ ii) $8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}} = \sqrt[3]{(8^2)} = \sqrt[3]{64} = 4$

b) Which of the calculations above is the easier method for evaluating $8^{\frac{2}{3}}$?

$(\sqrt[3]{8})^2$ is easier

c) Write the following in radical form and evaluate manually. Verify with a calculator.

i) $64^{\frac{3}{2}} = (\sqrt{64})^3 = 8^3 = \boxed{512}$ **ii)** $4^{\frac{5}{2}} = (\sqrt{4})^5 = 2^5 = \boxed{32}$ **iii)** $81^{\frac{3}{4}} = (\sqrt[4]{81})^3 = 3^3 = \boxed{27}$

3. a) Use exponent laws to simplify $8^{\frac{2}{3}} \times 8^{-\frac{2}{3}}$.

$8^{2/3+(-2/3)} = 8^0 = \boxed{1}$

b) Use the result in a) to write $8^{-\frac{2}{3}}$ in a form with a positive exponent. Evaluate $8^{-\frac{2}{3}}$ without using a calculator.

$8^{-2/3} = \dfrac{1}{8^{2/3}} = \dfrac{1}{(\sqrt[3]{8})^2} = \dfrac{1}{2^2} = \boxed{\dfrac{1}{4}}$

Rational Exponents

Simpler to evaluate in your head

$\boxed{a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m}$ or $a^{\frac{m}{n}} = \sqrt[n]{a^m}$, $m \in I$, $n \in N$, $a \neq 0$ when m is 0.

Note that if n is even, then a must be non-negative.

$\boxed{a^{-\frac{m}{n}} = \dfrac{1}{\left(\sqrt[n]{a}\right)^m}}$ or $a^{-\frac{m}{n}} = \dfrac{1}{\sqrt[n]{a^m}}$, $m \in I$, $n \in N$, $a \neq 0$ when m is 0.

Note that if n is even, then a must be positive.

Class Ex. #1

Write the following in radical form and evaluate without using a calculator. Verify with a calculator.

a) $25^{\frac{3}{2}} = (\sqrt{25})^3 = 5^3 = \boxed{125}$

b) $1000^{\frac{4}{3}} = (\sqrt[3]{1000})^4 = 10^4 = \boxed{10\,000}$

~~c)~~ $27^{-\frac{2}{3}}$

d) $16^{-\frac{3}{4}} = \dfrac{1}{16^{3/4}} = \dfrac{1}{(\sqrt[4]{16})^3} = \dfrac{1}{2^3} = \boxed{\dfrac{1}{8}}$

e) $(-8)^{\frac{2}{3}} = (\sqrt[3]{-8})^2 = (-2)^2 = \boxed{4}$

f) $-8^{\frac{2}{3}} = -(\sqrt[3]{8})^2 = -(2)^2 = \boxed{-4}$

~~g)~~ $(3^2 + 4^2)^{\frac{1}{2}}$

Class Ex. #2

Write the following in radical form and evaluate without using a calculator. Verify with a calculator.

a) $\left(\frac{9}{4}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{9}{4}}\right)^3 = \left(\frac{3}{2}\right)^3 = \boxed{\frac{27}{8}}$

b) $\left(\frac{9}{4}\right)^{-\frac{3}{2}} = \frac{1}{\left(\frac{9}{4}\right)^{3/2}} = \left(\frac{4}{9}\right)^{3/2} = \left(\sqrt{\frac{4}{9}}\right)^3 = \left(\frac{2}{3}\right)^3 = \boxed{\frac{8}{27}}$

Complete Assignment Questions #1 - #5

Class Ex. #3

Write an equivalent expression using radicals.

a) $r^{\frac{1}{3}} = \boxed{\sqrt[3]{r}}$ b) $s^{\frac{4}{7}} = \boxed{\left(\sqrt[7]{s}\right)^4}$ c) $t^{-\frac{1}{6}} = \frac{1}{t^{1/6}} = \boxed{\frac{1}{\sqrt[6]{t}}}$ d) $v^{-\frac{3}{2}} = \frac{1}{v^{3/2}} = \boxed{\frac{1}{\left(\sqrt{v}\right)^3}}$

Class Ex. #4

Consider the following powers.

A. $64^{\frac{2}{3}}$ $\left(\sqrt[3]{64}\right)^2$ **B.** $(-64)^{\frac{2}{3}}$ $\left(\sqrt[3]{-64}\right)^2$ **C.** $64^{\frac{3}{2}}$ $\left(\sqrt{64}\right)^3$ **D.** $(-64)^{\frac{3}{2}}$ $\left(\sqrt{-64}\right)^3$

Explain why three of the above powers can be calculated but the other has no meaning.

A, B, C can be evaluated. D cannot b/c it is the square root of a negative number

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Class Ex. #5

A cube has a volume of 60 m^3.

Do #1-12 (not 5, 9)

a) Write a power which represents the edge length of the cube.

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b) Write a power which represents the surface area of the cube.

c) Use a calculator to calculate the edge length and surface area to the nearest tenth.

Class Ex. #6

Write the number 10 in the following forms:

a) as a power with an exponent of $\frac{1}{2}$ b) as a power with an exponent of $\frac{1}{3}$

Complete Assignment Questions #6 - #13