

Exponents and Radicals Lesson #7: Rational Exponents - Part Two

Review Complete the following as a review.

Product Law $x^m x^n = x^{m+n}$

Quotient Law $x^m \div x^n = x^{m-n}$

Power of a Power $(x^m)^n = x^{m \cdot n}$

Power of a Product $(xy)^m = x^m y^m$

Power of a Quotient $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$, $y \neq 0$

Integral Exponent Rule $x^{-m} = \frac{1}{x^m}$, where $x \neq 0$

Rational Exponents $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $(\sqrt[n]{x})^m$

Writing Powers as Radicals

Class Ex. #1

Write each power as a radical.

a) $x^{\frac{1}{6}}$ b) $-y^{\frac{5}{4}}$ c) $(-z)^{\frac{5}{3}}$ d) $(-z)^{-\frac{5}{3}}$ e) $5t^{\frac{3}{4}}$ f) $(5t)^{\frac{3}{4}}$

Class Ex. #2

Simplify the following. Write each expression as a power with positive exponents and then as an entire radical.

a) $x^{\frac{3}{2}} \times x^{\frac{1}{2}} = x^{\frac{3}{2} + \frac{1}{2}} = x^{\frac{4}{2}} = x^2 = \sqrt{x^4}$

b) $y^{\frac{1}{3}} \div y^{\frac{5}{3}} = y^{\frac{1}{3} - \frac{5}{3}} = y^{-\frac{4}{3}} = \frac{1}{y^{\frac{4}{3}}} = \frac{1}{\sqrt[3]{y^4}}$

c) $(a^{\frac{1}{2}})^{\frac{2}{3}} = a^{\frac{1}{2} \cdot \frac{2}{3}} = a^{\frac{1}{3}} = \sqrt[3]{a}$

d) $\left(\frac{x^2}{y}\right)^{-\frac{1}{2}} = \left(\frac{y}{x^2}\right)^{\frac{1}{2}} = \frac{y^{\frac{1}{2}}}{x^{2 \cdot \frac{1}{2}}} = \frac{\sqrt{y}}{x}$

Class Ex. #3

Simplify the following. Write each expression as a power with positive exponents and then as an entire radical.

a) $4x^{\frac{3}{4}} \times 3x^{-\frac{1}{4}}$
 $= 12x^{\frac{3}{4} + (-\frac{1}{4})} = 12x^{\frac{2}{4}} = 12x^{\frac{1}{2}} = 12\sqrt{x}$

b) $5x^{\frac{3}{5}}$
 $= \frac{1}{5}x^{\frac{3}{5} - (-\frac{3}{5})} = \frac{1}{5}x^{\frac{6}{5}} = \frac{1}{5}\sqrt[5]{x^6}$

c) $(8a^{\frac{1}{2}})^{\frac{4}{3}}$
 $= 8^{\frac{4}{3}} a^{\frac{1}{2} \cdot \frac{4}{3}} = 8^{\frac{4}{3}} a^{\frac{2}{3}} = (\sqrt[3]{8})^4 a^{\frac{2}{3}} = 16a^{\frac{2}{3}} = 16\sqrt[3]{a^2}$

Complete Assignment Questions #1-#3

Writing Radicals as Powers

We can use the rule $a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$ to write radicals as powers.

Class Ex. #4

Write each radical as a power in the form a^n , $n \in \mathbb{Q}$.

a) $\sqrt[3]{a^5}$

b) $\sqrt[5]{a^2}$

c) $\sqrt{a^9}$

d) $\frac{1}{\sqrt{a^7}}$

Class Ex. #5

Write as a power and evaluate.

a) $\sqrt{\sqrt{1296}}$

$$= ((1296)^{1/2})^{1/2}$$

$$= 1296^{1/4}$$

$$= \boxed{6}$$

b) $\frac{1}{\sqrt{169}}$

$$= \frac{1}{169^{1/2}}$$

$$= \boxed{\frac{1}{13}}$$

c) $\sqrt[3]{\sqrt{64}}$

$$= ((64)^{1/2})^{1/3}$$

$$= 64^{1/6}$$

$$= \boxed{2}$$

Class Ex. #6

Write each expression in the form ax^n , where $a \in \mathbb{I}$, and $n \in \mathbb{Q}$. ← rational

a) $\sqrt[3]{8x^5}$

$$= \sqrt[3]{8} \cdot \sqrt[3]{x^5}$$

$$= \boxed{2x^{5/3}}$$

b) $\sqrt[5]{32x^3}$

$$= \sqrt[5]{32} \cdot \sqrt[5]{x^3}$$

$$= \boxed{2x^{3/5}}$$

c) $\sqrt{900x}$

$$= \sqrt{900} \cdot \sqrt{x}$$

$$= \boxed{30x^{1/2}}$$

d) $(\sqrt[3]{x^5})(\sqrt[3]{x})$

$$= x^{5/3} \cdot x^{1/3}$$

$$= x^{5/3 + 1/3}$$

$$= x^{6/3}$$

$$= \boxed{x^2}$$

e) $2\sqrt{x} \times \sqrt[3]{x}$

$$= 2x^{1/2} \cdot x^{1/3}$$

$$= 2x^{1/2 + 1/3}$$

$$= 2x^{3/6 + 2/6}$$

$$= \boxed{2x^{5/6}}$$

Class Ex. #7

Write an equivalent expression using exponents.

a) $\sqrt{\sqrt{a^3}}^{1/2}$
 $= ((a^3)^{1/2})^{1/2}$
 $= (a^3)^{1/4}$
 $= a^{3/4}$

b) $\sqrt[3]{64v^6}$
 $= \sqrt[3]{64} \sqrt[3]{v^6}$
 $= \sqrt[3]{4^3} (v^6)^{1/3}$
 $= 2(v^2)^{1/3} = 2\sqrt[3]{v^2}$

c) $(\sqrt[4]{x^5y^3})^{3/2}$
 $= ((x^5y^3)^{1/4})^{3/2}$
 $= (x^5y^3)^{3/8}$

Complete Assignment Questions #4 - #15

Assignment Do # 1-7, 14, 15 = $x^{15/8} y^{9/8}$

1. Write each power as an entire radical.

a) $a^{4/5}$ b) $b^{3/2}$ c) $c^{1/4}$ d) $x^{-2/5}$ e) $y^{-1/3}$

f) $5h^{2/3}$ g) $(5h)^{2/3}$ h) $-r^{5/4}$ i) $(-r)^{5/4}$ j) $2x^{-1/2}$

2. Simplify the following. Write each expression as a power with positive exponents and then as an entire radical.

a) $x^{7/2} \times x$ b) $y^{6/5} \div y^{4/5}$ c) $(a^{2/5})^3$ d) $(e^3f)^{3/2}$

e) $x^{1/2} \times x^{-1}$ f) $y^{2/7} \div y^{5/7}$ g) $\left(\frac{x}{y^4}\right)^{1/2}$ h) $\left(\frac{x^2}{y}\right)^{-3/2}$