Operations on Radicals Lesson #1:
Adding and Subtracting Radicals

Investigation 1

In the last unit we verified that addition of radicals cannot be done by adding the radicands.

In order to develop a rule for adding and subtracting radicals, complete the work below.

a) Use a calculator to investigate which of the following radical statements are true. Circle the statements which are true and place a single line through the expressions which are false.

i) $\sqrt{2} + 5\sqrt{2} = 6\sqrt{2}$ ✓

ii) $4\sqrt[3]{5} - 7\sqrt[3]{5} = -3\sqrt[3]{5}$ ✓

iii) $5\sqrt{8} - 2\sqrt{8} + 7\sqrt{8} = 10\sqrt{8}$ ✓

iv) $7\sqrt{5} + 7\sqrt[3]{5} = 14\sqrt[5]{5}$ ✗

v) $\sqrt[3]{3} + \sqrt[3]{2} = \sqrt[3]{5}$ ✗

b) Use the results in a) to suggest a rule for adding and subtracting radicals.

The index must match and the radicand must match. Add/subtract the coefficients.

c) Simplify the following. Express the answer as a mixed radical.

i) $8\sqrt{7} - 3\sqrt{7} + 15\sqrt{7}$ $= 20\sqrt{7}$

ii) $18\sqrt[5]{10} + 12\sqrt[5]{10} - 7\sqrt[5]{10}$ $= 23\sqrt[5]{10}$

iii) $5\sqrt{x} - 4\sqrt{x}$ $= \sqrt{x}$

Investigation 2

a) Use a calculator to verify that the following statements are true.

i) $\sqrt{2} + \sqrt{8} = 3\sqrt{2}$

ii) $5\sqrt{12} + 6\sqrt{48} = 34\sqrt{3}$

b) Does this appear to contradict the rule you wrote in Investigation #1 b)?

c) Complete the following by writing each radical in simplest mixed form to show that the rule can be modified.

i) $\sqrt{2} + \sqrt{8}$

$= \sqrt{2} +$

ii) $5\sqrt{12} + 6\sqrt{48}$

$=$

Adding and Subtracting Radicals

In order to add and subtract radicals, they must be able to be expressed as **like radicals**, i.e. radicals with the SAME radicand **and** the SAME index.

Class Ex. #1

Write each expression in terms of a single radical.

a) $\sqrt{80} - \sqrt{20}$

$= \sqrt{16 \cdot 5} - \sqrt{4 \cdot 5}$

$= 4\sqrt{5} - 2\sqrt{5}$

$= \boxed{2\sqrt{5}}$

b) $\sqrt[3]{80} + \sqrt[3]{270}$

$= \sqrt[3]{8 \cdot 10} + \sqrt[3]{27 \cdot 10}$

$= 2\sqrt[3]{10} + 3\sqrt[3]{10}$

$= \boxed{5\sqrt[3]{10}}$

c) $7\sqrt{27} - 3\sqrt{75} + 2\sqrt{147}$

$= 7\sqrt{9 \cdot 3} - 3\sqrt{25 \cdot 3} + 2\sqrt{49 \cdot 3}$

$= 21\sqrt{3} - 15\sqrt{3} + 14\sqrt{3}$

$= \boxed{20\sqrt{3}}$

Class Ex. #2

Simplify by combining like radicals.

a) $-5\sqrt{108} + \frac{3}{4}\sqrt{8} - \frac{5}{4}\sqrt{48} + \frac{1}{2}\sqrt{50}$

$= -5\sqrt{36 \cdot 3} + \frac{3}{4}\sqrt{4 \cdot 2} - \frac{5}{4}\sqrt{16 \cdot 3} + \frac{1}{2}\sqrt{25 \cdot 2}$

$= \boxed{-30\sqrt{3}} + \frac{3}{2}\sqrt{2} \boxed{-5\sqrt{3}} + \frac{5}{2}\sqrt{2}$

$= \boxed{-35\sqrt{3} + 4\sqrt{2}}$

b) $\frac{\sqrt[3]{64}}{8} + 2\sqrt[3]{375} - \frac{2\sqrt[3]{54}}{3} - \frac{5\sqrt[3]{24}}{2}$

$= \frac{4}{8} + 2\sqrt[3]{125 \cdot 3} - \frac{2}{3}\sqrt[3]{27 \cdot 2} - \frac{5}{2}\sqrt[3]{8 \cdot 3}$

$= \frac{1}{2} + 10\sqrt[3]{3} - 2\sqrt[3]{2} - 5\sqrt[3]{3}$

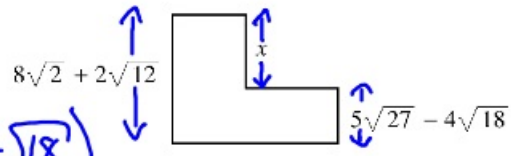
$= \boxed{\frac{1}{2} + 5\sqrt[3]{3} - 2\sqrt[3]{2}}$

Complete Assignment Questions #1 - #5

Class Ex. #3

Find the length of x

a) as an exact value
b) as a decimal to the nearest tenth



$x = (8\sqrt{2} + 2\sqrt{12}) - (5\sqrt{27} - 4\sqrt{18})$

$= 8\sqrt{2} + 2\sqrt{4 \cdot 3} - 5\sqrt{9 \cdot 3} + 4\sqrt{9 \cdot 2}$

$= 8\sqrt{2} + 4\sqrt{3} - 15\sqrt{3} + 12\sqrt{2}$

$= \boxed{20\sqrt{2} - 11\sqrt{3}} = 9.23\ldots = \boxed{9.2}$

Complete Assignment Questions #6 - #13

Assignment

Do #1-9 (not 4)
Quiz: L1

1. Simplify.

a) $5\sqrt{7} - 2\sqrt{7}$ **b)** $9\sqrt[3]{13} + 2\sqrt[3]{13}$ **c)** $4\sqrt{11} - 9\sqrt{11} + \sqrt{11}$

d) $4\sqrt{5} - 2\sqrt{2} + 8\sqrt{2}$ **e)** $13\sqrt[4]{a} + 7\sqrt[4]{a}$ **f)** $-3\sqrt{2} + 6\sqrt{3} - 9\sqrt{3} + 4\sqrt{2}$

2. Write each expression in terms of a single radical.

a) $\sqrt{125} - \sqrt{5}$ **b)** $\sqrt{27} + \sqrt{12}$ **c)** $\sqrt{24} - \sqrt{54} + 2\sqrt{6}$

d) $\sqrt{150} + \sqrt{216}$ **e)** $\sqrt[3]{16} + \sqrt[3]{128}$ **f)** $-3\sqrt{175} + 8\sqrt{28} - \sqrt{63}$

g) $\sqrt[4]{16} + \sqrt[4]{162}$ **h)** $2\sqrt{700} - 6\sqrt{63}$ **i)** $-7\sqrt[3]{54} - 2\sqrt[3]{250}$