

## Operations on Radicals Lesson #2: Multiplying Radicals

### Investigation

*Investigating Multiplication Properties of Radicals*

Use a calculator to determine whether the following statements are true or false.

a)  $\sqrt{2} \times \sqrt{3} = \sqrt{6}$     b)  $(2\sqrt{5})(-4\sqrt{3}) = -8\sqrt{15}$     c)  $\sqrt{2} \cdot \sqrt[3]{4} = \sqrt[3]{8}$

d)  $2\sqrt[3]{10} \times 3\sqrt[3]{7} = 6\sqrt[3]{70}$     e)  $(4\sqrt[3]{5})(7\sqrt{6}) = 28\sqrt[6]{30}$

Based on the results from a) - c), write a rule which describes the process of multiplying radicals.

### Multiplying Radicals

To multiply radicals, the index must be the same in each radical.



- Multiply numerical coefficients by numerical coefficients.
- Multiply radicand by radicand.
- Simplify into mixed radical form if possible.



It is usually easier to convert each radical to its simplest mixed form before multiplying.

*Class Ex. #1*

Multiply and simplify where possible.

a)  $\sqrt{8} \cdot \sqrt{8}$     b)  $(4\sqrt{5})(3\sqrt{6})$     c)  $(4\sqrt{x})(3\sqrt{y})$     d)  $-2\sqrt{8} \times 5\sqrt{12}$

$= \sqrt{64}$   
 $= \boxed{8}$

$= \boxed{12\sqrt{30}}$

$= \boxed{12\sqrt{xy}}$

$= -2\sqrt{4 \cdot 2} \times 5\sqrt{4 \cdot 3}$   
 $= -4\sqrt{2} \times 10\sqrt{3}$   
 $= \boxed{-40\sqrt{6}}$

Class Ex. #2

Expand and simplify.

a)  $\sqrt{5}(2\sqrt{10} - \sqrt{5})$

$$= 2\sqrt{50} - \sqrt{25}$$

$$= 2\sqrt{25 \cdot 2} - 5$$

$$= \boxed{10\sqrt{2} - 5}$$

b)  $2\sqrt{5}(3\sqrt{45} - 8\sqrt{5} + 3\sqrt{20})$

$$= 2\sqrt{5}(9\sqrt{5} - 8\sqrt{5} + 6\sqrt{5})$$

$$= 2\sqrt{5}(7\sqrt{5})$$

$$= 14\sqrt{25} = 14(5) = \boxed{70}$$

c)  $2(\sqrt{3} - \sqrt{5}) - \sqrt{2}(\sqrt{6} + \sqrt{10})$

$$= 2\sqrt{3} - 2\sqrt{5} - \sqrt{12} - \sqrt{20}$$

$$= \boxed{2\sqrt{3}} - \boxed{2\sqrt{5}} - \boxed{2\sqrt{3}} - \boxed{2\sqrt{5}}$$

$$= \boxed{-4\sqrt{5}}$$

d)  $-4\sqrt{a}(\sqrt{a} - 9\sqrt{b})$

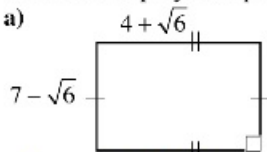
$$= -4\sqrt{a^2} + 36\sqrt{ab}$$

$$= \boxed{-4a + 36\sqrt{ab}}$$

Class Ex. #3

Write and simplify an expression for the area of each shape.

a)



$$A = (4 + \sqrt{6})(7 - \sqrt{6})$$

$$= 28 - 4\sqrt{6} + 7\sqrt{6} - \sqrt{36}$$

$$= 28 + 3\sqrt{6} - 6$$

$$= \boxed{22 + 3\sqrt{6}}$$

b)  $2\sqrt{18} - \sqrt{27}$



Complete Assignment Questions #1 - #10

**Multiplying Conjugate Binomials**

Expand the following expressions:

i)  $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

$$= \sqrt{25} + \sqrt{10} - \sqrt{10} - \sqrt{4}$$

$$= 5 - 2 = \boxed{3}$$

ii)  $(2\sqrt{7} + 8)(2\sqrt{7} - 8)$

$$= 4\sqrt{49} - 16\sqrt{7} + 16\sqrt{7} - 64$$

$$= 4(7) - 64 = 28 - 64$$

$$= \boxed{-36}$$

The pairs of binomials above are called **conjugates** of each other.

What do you notice about the product of two conjugate binomials?

No radicals.



- Conjugate binomials are pairs of binomials in the form  $a\sqrt{b} + c\sqrt{d}$  and  $a\sqrt{b} - c\sqrt{d}$ .
- The product of conjugate binomials is always a rational number of the form  $a^2b - c^2d$ .



Write the conjugate of each, then multiply each pair.

a)  $4\sqrt{6} + 3$

b)  $-3\sqrt{11} + \sqrt{2}$

c)  $5\sqrt{x} - \sqrt{y}$

$$\begin{aligned}
 & 4\sqrt{6} - 3 \\
 & = (4\sqrt{6} + 3)(4\sqrt{6} - 3) \\
 & = 16\sqrt{36} - 12\sqrt{6} + 12\sqrt{6} - 9 \\
 & = 16(6) - 9 = 96 - 9 = \boxed{87}
 \end{aligned}$$

$$\begin{aligned}
 & 5\sqrt{x} + \sqrt{y} \\
 & = (5\sqrt{x} - \sqrt{y})(5\sqrt{x} + \sqrt{y}) \\
 & = 25\sqrt{x^2} + \cancel{5\sqrt{xy}} - \cancel{5\sqrt{xy}} - \sqrt{y^2} \\
 & = \boxed{25x - y}
 \end{aligned}$$

Complete Assignment Questions #11 - #17

## Assignment

Do # 1-13 (a, c, e...) (all of S)

1. Multiply and simplify where possible. Do not use a calculator.

a)  $(\sqrt{7})(\sqrt{3})$     b)  $4\sqrt{3} \times 2\sqrt{5}$     c)  $-3\sqrt{5} \times 2\sqrt{2}$     d)  $6\sqrt{p} \times 8\sqrt{q}$

e)  $(\sqrt{15})(\sqrt{3})$     f)  $10\sqrt{5} \times 9\sqrt{5}$     g)  $3\sqrt{6} \cdot 5\sqrt{10}$     h)  $\sqrt{a} \times 10\sqrt{a}$

i)  $7\sqrt{54} \cdot 2\sqrt{6}$     j)  $(\sqrt{32})(\sqrt{6})$     k)  $\sqrt{15} \times 3\sqrt{27}$