Operations on Radicals Lesson #3:
Dividing Radicals - Part One

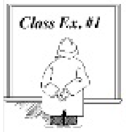
Dividing Radicals

In previous work, we discovered that $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$, $a \geq 0,\ b > 0$, and $a, b \in R$.

We can use this rule to divide radicals of the form $\dfrac{m\sqrt{a}}{n\sqrt{b}}$.

To divide radicals, the index must be the same in each radical.

- Divide numerical coefficients by numerical coefficients.
- Divide radicand by radicand.
- Simplify into mixed radical form if possible.



Divide and simplify where possible.

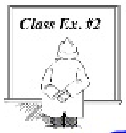
a) $\dfrac{\sqrt{30}}{\sqrt{6}}$ $=\boxed{\sqrt{5}}$

b) $\dfrac{8\sqrt[3]{21}}{2\sqrt[3]{3}}$ $=\boxed{4\sqrt[3]{7}}$

c) $\dfrac{15\sqrt{48}}{10\sqrt{6}}$ $=\dfrac{3}{2}\sqrt{8}=\dfrac{3}{2}\sqrt{4\cdot2}=\boxed{3\sqrt{2}}$

d) $\dfrac{4\sqrt{ab}}{12\sqrt{a}}$ $=\boxed{\dfrac{1}{3}\sqrt{b}}$

In some cases, converting a radical into its simplest mixed radical form before dividing will make the calculation easier.



Simplify numerator and denominator, then divide.

a) $\dfrac{4\sqrt{54}}{3\sqrt{8}}$ $=\dfrac{4\sqrt{9\cdot6}}{3\sqrt{4\cdot2}}=\dfrac{12\sqrt{6}}{6\sqrt{2}}=\boxed{2\sqrt{3}}$

b) $\dfrac{8\sqrt{126}}{\sqrt{112}}$ $=\dfrac{8\sqrt{9\cdot14}}{\sqrt{16\cdot7}}=\dfrac{24\sqrt{14}}{4\sqrt{7}}=\boxed{6\sqrt{2}}$

c) $\dfrac{10\sqrt[3]{162}}{20\sqrt[3]{128}}$ $=\dfrac{10\sqrt[3]{27\cdot6}}{20\sqrt[3]{64\cdot2}}=\dfrac{30\sqrt[3]{6}}{80\sqrt[3]{2}}=\boxed{\dfrac{3}{8}\sqrt[3]{3}}$

Divide each term in the numerator by the denominator, and simplify.

$$\frac{\sqrt{24}+\sqrt{48}-\sqrt{108}}{\sqrt{6}}$$

$$=\frac{\sqrt{24}}{\sqrt{6}}+\frac{\sqrt{48}}{\sqrt{6}}-\frac{\sqrt{108}}{\sqrt{6}}=\sqrt{4}+\sqrt{8}-\sqrt{18}$$
$$=2+2\sqrt{2}-3\sqrt{2}$$
$$=\boxed{2-\sqrt{2}}$$

Complete Assignment Questions #1 - #4

Rationalizing the Denominator

Usually answers are written in **simplest form**, e.g. $\frac{1}{6}+\frac{1}{3}=\frac{3}{6}$ which simplifies to $\frac{1}{2}$.

In the division of radicals in this unit, regard simplest form as the form in which

i) the denominator of the fraction is a rational number, i.e. it does not contain a radical
ii) the radicand cannot contain a fraction and is expressed in simplest mixed form

The process of eliminating the radical from the denominator (i.e. converting the denominator from an irrational number to a rational number) is called **rationalizing the denominator**. The denominators in this lesson are all of monomial form. Denominators in binomial form will be discussed in the next lesson.

Class Ex. #4

Simplify by rationalizing the denominator.

a) $\frac{1}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}}=\frac{\sqrt{13}}{\sqrt{169}}=\boxed{\frac{\sqrt{13}}{13}}$

b) $\frac{\sqrt{5}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\boxed{\frac{\sqrt{10}}{2}}$

c) $\frac{\sqrt{2}}{-\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}}=\frac{\sqrt{12}}{-6}=\frac{2\sqrt{3}}{-6}=\boxed{\frac{-\sqrt{3}}{3}}$

d) $\frac{\sqrt{20}}{\sqrt{3}}=\frac{2\sqrt{5}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\boxed{\frac{2\sqrt{15}}{3}}$

Class Ex. #5

Simplify.

a) $\frac{7}{3\sqrt{7}}$ **b)** $\sqrt{\frac{18}{5}}$ **c)** $\frac{3\sqrt{12}}{\sqrt{72}}$

Class Ex. #6

Simplify the radical expression $\frac{3\sqrt{18}-\sqrt{12}}{\sqrt{2}}$ by

a) rationalizing the denominator

$=\frac{(3\sqrt{18}-\sqrt{12})}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$=\frac{3\sqrt{36}-\sqrt{24}}{2}$

$=\frac{\cancel{18}^{9}-\cancel{2}^{1}\sqrt{6}}{\cancel{2}_{1}}$

$=\boxed{9-\sqrt{6}}$

b) dividing numerator and denominator by $\sqrt{2}$

$=\frac{3\sqrt{18}}{\sqrt{2}}-\frac{\sqrt{12}}{\sqrt{2}}$

$=3\sqrt{9}-\sqrt{6}$

$=\boxed{9-\sqrt{6}}$

Do
#1-7 (a,c,e....)
8-13
Quiz: L2+3

Complete Assignment Questions #5 - #16