Factoring and Applications Lesson #2:
Common Factors and Grouping

Binomial Common Factors

In certain circumstances, the greatest common factor may be a binomial rather than a monomial. This particular type of factoring is part of a process for factoring trinomials of the form $ax^2 + bx + c$, where $a \neq 1$, and is covered in the next lesson.

Class Ex. #1

Factor the following polynomials by removing the greatest common factor.

a) $(4x)(x + 7) - (3)(x + 7)$ $= (x+7)(4x-3)$

b) $7(3 - 2y) + 2y(3 - 2y)$ $= (3-2y)(7+2y)$

c) $9a(4a + 1) + 1(4a + 1)$ $= (4a+1)(9a+1)$

Class Ex. #2

Factor the following and write the answer in simplest factored form.

a) $(3y + 2)(5y + 1) + (3y + 2)(4y)$

$= (3y+2)((5y+1)+4y)$

$= \boxed{(3y+2)(9y+1)}$

b) $3a(a - 6) - 9(a - 6)$

$= (a-6)(3a-9)$

$= (a-6)3(a-3) = \boxed{3(a-6)(a-3)}$

c) $2x(x - 5) + 5(5 - x)$

$= 2x(x-5) + 5(-1)(-5+x)$

$= 2x(x-5) - 5(x-5) = \boxed{(x-5)(2x-5)}$

d) $20x(x - 3) - 4(3 - x)$

Complete Assignment Question #1

Factoring by Grouping

Sometimes polynomials in four terms can be factored by removing the greatest common factor from a pair of terms followed by a binomial common factor. This method is called factoring by grouping. The method of grouping is a component of the method used to factor trinomials of the form $ax^2 + bx + c$, where $a \neq 1$, and is covered in the next lesson.

Class Ex. #3

Factor the following polynomials by grouping.

a) $\underbrace{x^2 + 3x} + \underbrace{6x + 18}$

$= x(x+3) + 6(x+3)$

$= \boxed{(x+3)(x+6)}$

b) $8x^2 - 2x + 12x - 3$

c) $\underbrace{8a^2 - 4a} - \underbrace{10a + 5}$

$= 4a(2a-1) - 5(2a-1)$

$= \boxed{(2a-1)(4a-5)}$

d) $6a^2 - 9a - 2a + 3$

$= 3a(2a-3) - (2a-3)$

$= \boxed{(2a-3)(3a-1)}$

e) $pq + pr - sq - sr$

$= p(q+r) - s(q+r)$

$= \boxed{(q+r)(p-s)}$

f) $5x^2 + 18y^2 - 15xy^2 - 6x$

$= 5x^2 - 6x + 18y^2 - 15xy^2$

$= x(5x-6) - 3y^2(-6+5x)$

$= \boxed{(5x-6)(x-3y^2)}$

Complete Assignment Questions #2 - #4

Monomial Common Factors involving Fractions

In polynomials involving fractional coefficients, it is useful to include a fraction as part of the monomial common factor so that the remaining factor is an integral polynomial with no common factor.

eg. $\frac{1}{2}x^2 - 3x = \frac{1}{2}x(x - 6)$

Such a technique will prove useful in future math courses.

Class Ex. #4

In each case, a common factor has been removed so that the remaining factor is an integral binomial. Complete the factoring and check mentally by expanding the factored form.

a) $\frac{1}{3}x^2 + 4x = \frac{1}{3}x(x + 12)$

$\frac{1}{3}x \cdot ? = 4x \to ? = 4x \div \frac{1}{3}x = 4x \cdot \frac{3}{x} = 12$

b) $\frac{1}{4}a^2 - 4a = \frac{1}{4}a(a - 16)$

$-4a \div \frac{1}{4}a = -4a \cdot \frac{4}{a} = -16$

c) $6x + \frac{2}{3} = \frac{2}{3}(9x + 1)$

$6x \div \frac{2}{3} = 6x \cdot \frac{3}{2} = 9x$

d) $\frac{2}{4}a^2 - \frac{3}{4}b^2 = \frac{1}{4}(2a^2 - 3b^2)$

Class Ex. #5

Complete the factoring and check mentally by expanding the factored form.

a) $a - \frac{1}{6}a^2 = \frac{1a}{6}(6 - a)$

b) $\frac{1}{2}\pi r^2 - 2\pi r = \frac{\pi r}{2}(r - 4)$

c) $4x^2 + 2x + \frac{2}{5} = \frac{2}{5}(10x^2 + 5x + 1)$

$2x \div \frac{2}{5} = 2x \cdot \frac{5}{2} = 5x$

Class Ex. #6

In each case, remove a common factor so that the remaining factor is an integral polynomial.

a) $\frac{8}{4}a + \frac{1}{4}a^3$

$= \frac{1}{4}a(8 + a^2)$

b) $\frac{6}{6}x^3 + \frac{2}{6}x^2 - \frac{1}{6}x$

$= \frac{1}{6}x(6x^2 + 2x - 1)$

Complete Assignment Questions #5 - #14

Do #1-4 (a,c,e...)
5-9 (not 8)

Quiz: L2 Friday