Factoring and Applications Lesson #4:
Factoring Trinomials of the Form $a(f(x))^2 + b(f(x)) + c$

In the previous lessons we have factored

- trinomials of the form $x^2 + bx + c$ by inspection
- trinomials of the form $ax^2 + bx + c$, by decomposition

In this lesson, we extend this process to consider expressions in which the variable x is replaced by a function of x.

Factoring Trinomials of the form $(f(x))^2 + b(f(x)) + c$ where $f(x)$ is a Monomial

The method of inspection can be extended to factor polynomial expressions of the form $(f(x))^2 + b(f(x)) + c$, where $f(x)$ itself is a polynomial.
In this section, we will restrict f to be a monomial.

In the trinomial $x^2 + bx + c$, the degrees of the terms are 2, 1, and 0 respectively. The method of inspection can also be used when the terms have degrees 4, 2, and 0 or 6, 3, and 0 etc.

In all cases, we make a substitution which results in a trinomial with terms of degree 2, 1, and 0.

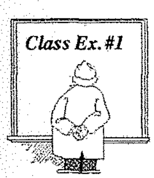
The following example to factor $x^4 + 5x^2 + 6$ illustrates the process.

$x^4 + 5x^2 + 6$ can be written $(x^2)^2 + 5(x^2) + 6$.

Make the substitution $A = x^2$ so the expression becomes $A^2 + 5A + 6$, which factors to $(A + 2)(A + 3)$.

Replace A by x^2 to get $(x^2 + 2)(x^2 + 3)$. Then $x^4 + 5x^2 + 6 = (x^2 + 2)(x^2 + 3)$.

With experience, this process can be done by inspection.
Note that in this example the function $f(x) = x^2$.



Factor completely.

a) $a^4 - 5a^2 - 14$

Let $A = a^2$

$A^2 - 5A - 14$

$= (A - 7)(A + 2)$

$= (a^2 - 7)(a^2 + 2)$

b) $x^4 + 4x^2 - 5$

Let $A = x^2$

$A^2 + 4A - 5$

$= (A - 1)(A + 5)$

$= (x^2 - 1)(x^2 + 5)$

$= (x - 1)(x + 1)(x^2 + 5)$

c) $x^6 - 9x^3 + 14$

Let $A = x^3$

$A^2 - 9A + 14$

$= (A - 2)(A - 7)$

$= (x^3 - 2)(x^3 - 7)$

Factoring Trinomials of the form $a(f(x))^2 + b(f(x)) + c$ *where f(x) is a Monomial*

The method of decomposition can be extended to factor polynomial expressions of the form $af(x)^2 + b(f(x)) + c$ where f itself is a polynomial.
In this section, we will restrict f to be a monomial.

In the trinomial $ax^2 + bx + c$, the degrees of the terms are $2, 1,$ and 0 respectively.
The method of decomposition can also be used when the terms have degrees $4, 2,$ and 0 or $6, 3,$ and 0 etc.

The expression $4y^4 - 11y^2 - 3$ can be factored using the method of decomposition by substituting $A = y^2$ or by splitting $-11y^2$ into two terms in y^2.

Complete the work started below.

Method 1

$4y^4 - 11y^2 - 3 = 4(y^2)^2 - 11(y^2) - 3$

Let $A = y^2$ $\qquad 4A^2 - 11A - 3$

$= 4A^2 - 12A + A - 3$

$= 4A(A-3) + 1(A-3)$

$= (A-3)(4A+1)$

$= (y^2-3)(4y^2+1)$

Method 2

$4y^4 - 11y^2 - 3 = 4y^4 - 12y^2 + 1y^2 - 3$

$= 4y^2(y^2-3) + 1(y^2-3)$

$= (y^2-3)(4y^2+1)$

Class Ex. #2

Factor completely.

a) $4x^4 - 5x^2 - 6$

×	+
-24	-5

$(-8, 3)$

$= 4x^4 - 8x^2 + 3x^2 - 6$

$= 4x^2(x^2-2) + 3(x^2-2)$

$= (x^2-2)(4x^2+3)$

b) $2a^2b^2 - 31ab + 99$

×	+
198	-31

$(-22, -9)$

$= 2a^2b^2 - 22ab - 9ab + 99$

$= 2ab(ab-11) - 9(ab-11)$

$= (ab-11)(2ab-9)$

Class Ex. #3

Factor completely the expression $8x^4 + 10x^2 - 3$.

×	+
-24	10

$(-2, 12)$

$8x^4 + 10x^2 - 3 = 8x^4 - 2x^2 + 12x^2 - 3$

$= 2x^2(4x^2-1) + 3(4x^2-1)$

$= (4x^2-1)(2x^2+3)$

$= (2x-1)(2x+1)(2x^2+3)$

Class Ex. #4

Given that $(\sin x)^2$ is written as $\sin^2 x$ and $(\cos x)^2$ is written as $\cos^2 x$, factor

a) $6\sin^2 x - 7\sin x + 2$

$= 6\sin^2 x - 3\sin x - 4\sin x + 2$

$= 3\sin x(2\sin x - 1) - 2(2\sin x - 1)$

$= (2\sin x - 1)(3\sin x - 2)$

b) $4\cos^2 x + 11\cos x - 3$

$= 4\cos^2 x - 1\cos x + 12\cos x - 3$

$= \cos x(4\cos x - 1) + 3(4\cos x - 1)$

$= (4\cos x - 1)(\cos x + 3)$

Complete Assignment Questions #1 - #4

Factoring Trinomials of the form $a(f(x))^2 + b(f(x)) + c$ ***where f(x) is a Binomial***

Class Ex. #5

Factor.

a) $7(x-3)^2 - 4(x-3) - 3$

Let $A = x - 3$

$7A^2 - 4A - 3$

$= 7A^2 - 7A + 3A - 3$

$= 7A(A-1) + 3(A-1)$

$= (A-1)(7A+3)$

$= ((x-3)-1)(7(x-3)+3)$

$= (x-4)(7x-21+3)$

$= (x-4)(7x-18)$

b) $9(a+4)^2 + (a+4) - 10$

Let $A = a + 4$

$9A^2 + A - 10$

$= 9A^2 - 9A + 10A - 10$

$= 9A(A-1) + 10(A-1)$

$= (A-1)(9A+10)$

$= ((a+4)-1)(9(a+4)+10)$

$= (a+3)(9a+36+10)$

$= (a+3)(9a+46)$

Complete Assignment Questions #5 - #10