Factoring and Applications Lesson #5:
Factoring $a^2x^2 - b^2y^2$ and $a^2(f(x))^2 - b^2(f(x))^2$

Factoring $a^2x^2 - b^2y^2$

Complete the following:

- The expanded form of $(ax - by)(ax + by)$ is $a^2x^2 - b^2y^2$.
- The factored form of $a^2x^2 - b^2y^2$ is $(ax+by)(ax-by)$

Class Ex. #1

Factor:

a) $9x^2 - 16y^2$ $=(3x+4y)(3x-4y)$

b) $3x^3y - 27xy^3$ $=3xy(x^2-9y^2)$ $=3xy(x+3y)(x-3y)$

c) $144p^2q^2 - 4r^2$ $=4(36p^2q^2-r^2)$ $=4(6pq+r)(6pq-r)$

Class Ex. #2

A Longhouse was the traditional communal dwelling of the Squamish peoples. Its innovative rectangular design allow for this dwelling to be assembled and taken a part as needed in order to be transported to seasonal locations. Today a Longhouse is often a meaningful place to come together for ceremonies, political meetings and community gatherings such as a potlatch. https://www.thecanadianencyclopedia.ca/en/article/longhouse

A Longhouse has been disassembled and will need to moved to a new location. Its footprint is rectangular with an area of $49x^2 - 16y^2$ and its length is double its width.

a) The length and width of the Longhouse can each be written in the form $(ax + by)$ where a and b are integers. Determine appropriate values for a and b, and hence write expressions in x and y for the length and width of the Longhouse.

$A = 49x^2 - 16y^2$

$w = (7x+4y)(7x-4y)$

Length $= 7x+4y$

Width $= 7x-4y$

b) A Longhouse with a perimeter of 56 m is reassembled.
Given that the length of the Longhouse is double its width, develop a strategy for determining the length and width of the ~~pane~~ longhouse.

$l = 2w$

$7x+4y = 2(7x-4y)$

$7x+4y = 14x-8y$

$0 = 7x-12y$ ①

$2(l+w) = 56$

$2(7x+4y+7x-4y) = 56$ ②

$2(14x) = 56$

$28x = 56$

$x = 2$

Complete Assignment Questions #1 - #2

$x = 2$ in ①

$0 = 7(2) - 12y$

$-14 = -12y$

$\frac{7}{6} = y$

$\therefore$ Length $= 7(2) + 4\left(\frac{7}{6}\right) = 18\frac{2}{3}$ m

Width $= 7(2) - 4\left(\frac{7}{6}\right) = 9\frac{1}{3}$ m

Factoring $a^2(f(x))^2 - b^2(g(y))^2$ where $f(x)$ and $g(y)$ are Monomials

The method of difference of squares in which $a^2 - b^2 = (a-b)(a+b)$ can also be extended to include examples where a and b represent polynomials.

The following process can be used to factor $x^4 - 16y^4$.

$x^4 - 16y^4$ can be written $(x^2)^2 - (4y^2)^2$.

Make the substitution $A = x^2$ and $B = 4y^2$ so the expression becomes $A^2 - B^2$ which factors to $(A-B)(A+B)$.

Replace A by x^2 and B by $4y^2$ to get $(x^2 - 4y^2)(x^2 + 4y^2)$, which factors further to $(x-2y)(x+2y)(x^2+4y^2)$.

In this example $f(x) = x^2$ and $g(y) = y^2$.

Factor completely.

a) $k^4 - 1$

$= (k^2+1)(k^2-1)$

$= (k^2+1)(k+1)(k-1)$

~~b)~~ $80a^4 - 5x^4$

c) $2p^5q^4 - 162pt^4$

$= 2p(p^4q^4 - 81t^4)$

$= 2p(p^2q^2 + 9t^2)(p^2q^2 - 9t^2)$

$= 2p(p^2q^2 + 9t^2)(pq + 3t)(pq - 3t)$

Complete Assignment Questions #3 - #4

Factoring $a^2(f(x))^2 - b^2(g(y))^2$ where $f(x)$ and/or $g(y)$ are Binomial(s)

Factor completely.

a) $a^2 - (b-c)^2$

b) $(2x-y)^2 - (x+y)^2$

$= ((2x-y) + (x+y))((2x-y) - (x+y))$

$= (3x)(x-2y)$

Factor the expression $36(x+5)^2 - 49(x-8)^2$.

$= (6(x+5)+7(x-8))(6(x+5)-7(x-8))$

$= (6x+30+7x-56)(6x+30-7x+56)$

$= (13x-26)(-x+86)$

$= 13(x-2)(-x+86)$

Complete Assignment Questions #5 - #11

Assignment

Do #1,3,4,5 (a,c,e...) all of 2,6

Quiz: 4+5

1. Factor.

a) $16x^2 - 49y^2$ **b)** $25a^2 - 121y^2$ **c)** $p^2q^2 - r^2s^2$

d) $16x^2 - 4y^2$ **e)** $9a^2b^2 - 36c^2$ **f)** $12a^2 - 75p^2q^2$

g) $4xy^3 - 169x^3y$ **h)** $60a^2b^2 - 15a^4b^4$ **i)** $4b^2g^2 - 49t^2z^2$

j) $25x^2 + 100y^2$ **k)** $225a^2c^2 - 16b^2d^2$ **l)** $xw^2y^2 - x^3z^2$

m) $1 - \cos^2 x$ **n)** $\sin^2 x - \cos^2 x$ **o)** $\frac{x^2}{64} - \frac{y^2}{49}$