

Factoring and Applications Lesson #6: Solving Quadratic Equations Using Factoring

In this lesson, we introduce an application of factoring, namely the solution to polynomial equations and in particular, quadratic equations. Some students may have covered this lesson as an enrichment in the *Foundations of Mathematics and Pre-Calculus Grade 10 Book*.

Investigating the Zero Product Law

Complete the following:

- The statement $x - 3 = 0$ is true only if $x = \underline{3}$.
- The statement $x + 1 = 0$ is true only if $x = \underline{-1}$.
- The statement $(x - 3)(x + 1) = 0$ is true if $x = \underline{3}$ or if $x = \underline{-1}$.
- The statement $x(x + 1) = 0$ is true if $\underline{x=0 \text{ or } x=-1}$

The Zero Product Law

The last two statements in the investigation above are examples of what is called The Zero Product Law which states the following:

If the **product** of multiple factors is **equal to zero**, then at least **one of the factors must be equal zero**.

- Complete. If $a \times b = 0$, then $a = \underline{0}$ or $b = \underline{0}$.

• Complete the following by solving for x in each equation:

a) $\frac{8x}{8} = 0$ b) $\frac{8(x+2)(x-7)}{8} = 0$ c) $x(x+2)(x-7) = 0$ d) $(2x+1)(3x-2) = 0$

$$\boxed{x=0}$$

$$\begin{aligned} (x+2)(x-7) &= 0 \\ x &= -2 \\ x &= 7 \end{aligned}$$

$$\begin{aligned} x &= 0 \\ x &= -2 \\ x &= 7 \end{aligned}$$

$$\begin{aligned} 2x+1 &= 0 \\ 2x &= -1 \\ x &= -\frac{1}{2} \end{aligned}$$

$$3x-2=0$$

$$\begin{aligned} 3x &= 2 \\ x &= \frac{2}{3} \end{aligned}$$

e) Why is it acceptable to divide by 8 first in (a) and (b)?

Dividing by 8 on each side doesn't change the equation

f) Why is it **not** acceptable to divide by x first in (c)?

We can't divide by x because $x=0$ and division by zero is not defined.

All the above equations are polynomial equations in which one side is a polynomial expression and the **other side equals zero**.

The **solution** to a polynomial equation is given by stating the value(s) of the variable which make(s) the left side and the right side equal. These values are said to **satisfy** the equation.

We can use the Zero Product Law to solve polynomial equations of various types. In this lesson, we will focus on quadratic equations using factoring and the zero product law.

Solving Quadratic Equations

Consider the equation $x^2 - 2x - 3 = 0$. Factoring the left side leads to $(x - 3)(x + 1) = 0$. This is true if $x = 3$ or if $x = -1$. Since the equation is satisfied by both $x = 3$ and $x = -1$, the solutions to the equation are $x = 3$ **and** $x = -1$, sometimes written as $x = -1, 3$.

Class Ex. #1



Complete the solution to the equation $x^2 - 9x + 20 = 0$.

$$x^2 - 9x + 20 = 0$$

$$(x - 4)(x - 5) = 0$$

$$x - 4 = 0 \text{ or } x - 5 = 0$$

$$x = 4 \text{ or } x = 5$$

The solutions are $x = 4$ and $x = 5$

or $x = 4, 5$

Class Ex. #2



Solve the equation by using the Zero Product Law.

a) $x^2 - 81 = 0$

$$(x - 9)(x + 9) = 0$$

$$x = 9, -9$$

b) $4x^2 - 9 = 0$

$$(2x + 3)(2x - 3) = 0$$

$$x = -\frac{3}{2}, \frac{3}{2}$$

c) $10x^2 - 90x = 0$

$$10x(x - 9) = 0$$

$$x = 0, 9$$

d) $10x^2 - 90 = 0$

$$10(x^2 - 9) = 0$$

$$10(x + 3)(x - 3) = 0$$

$$x = \pm 3$$

Class Ex. #3



Solve the equation.

a) $3x^2 - 13x - 10 = 0$

$$3x^2 - 15x + 2x - 10 = 0$$

$$3x(x - 5) + 2(x - 5) = 0$$

$$(x - 5)(3x + 2) = 0$$

$$x = 5, -\frac{2}{3}$$

b) $5x^2 + 30x = -25$

$$5x^2 + 30x + 25 = 0$$

$$5(x^2 + 6x + 5) = 0$$

$$5(x + 1)(x + 5) = 0$$

$$x = -1, -5$$

$$\begin{array}{r} x \mid + \\ 5 \mid 6 \\ \hline \hookrightarrow 1, 5 \end{array}$$

Complete Assignment Questions #1 - #4

Problem Solving with Quadratic Equations

Some problems in mathematics can be solved by the following procedure:

- i) Introduce a variable to represent an unknown value.
- ii) Form a quadratic equation from the given information.
- iii) Solve the quadratic equation using the methods in this lesson.
- iv) State the solution to the problem.

In this section we will consider fairly routine problems. This topic will be extended in a higher level math course.



Class Ex. #4

The area of a rectangular sheet of paper is 300 cm^2 . The length is 5 cm more than the width. Form a polynomial equation and solve it to determine the perimeter of the rectangular sheet.

Area = 300 cm^2

$A = lw$

$300 = (w+5)w$

$300 = w^2 + 5w$

$0 = w^2 + 5w - 300$

$0 = (w-15)(w+20)$

$w = 15 \text{ cm}$ (not 20 cm)

$l = 15 + 5$

$l = 20 \text{ cm}$

Do # 1-7 (not 6)

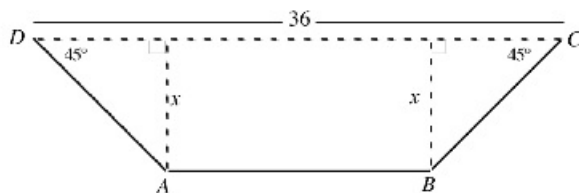
no -ve dimensions

$x \mid +$
 $-300 \mid 5$
 $\hookrightarrow -15, 20$



Class Ex. #5

The diagram shows the cross-section of a water trough whose sloping sides AD and BC make an angle of 45° with the horizontal. The length $DC = 36 \text{ cm}$.



- a) Show that the area of the cross-section is $x(36 - x) \text{ cm}^2$.