

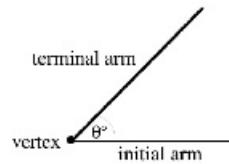
Trigonometry - Angles and Ratios Lesson #1: Rotation Angles and Reference Angles

Angles in Standard Position

Angles can be measured in degrees where 360° is one complete rotation.

A **rotation angle** is formed by rotating an **initial arm** (or initial side) through an angle θ° about a fixed point (the vertex).

The angle formed between the initial arm and the terminal arm (or terminal side) is the rotation angle.



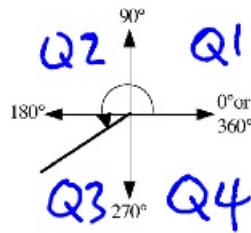
A **positive angle** results from a counter clockwise rotation.

A **negative angle** results from a clockwise rotation.

The angle shown in the above diagram is said to be in **standard position**.

On a coordinate grid, standard position means the initial arm is along the positive x -axis and the rotation is about the origin.

The diagram below shows an angle of 220° in standard position.

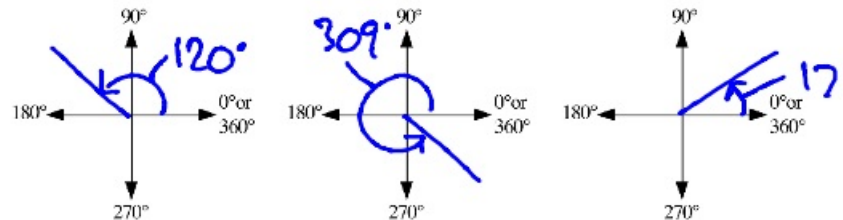


Class Ex. #1



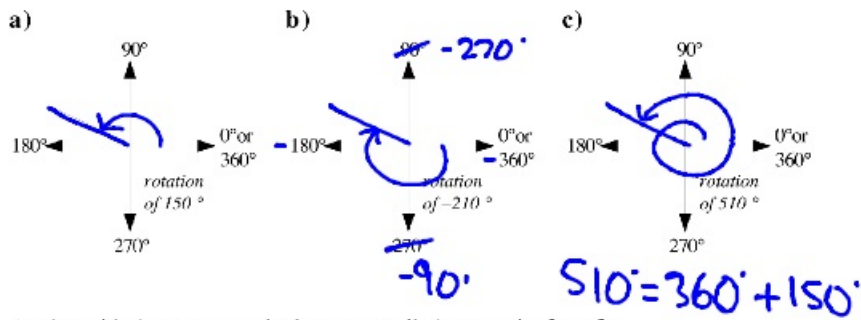
Sketch the rotation angle in standard position and state the quadrant in which the angle terminates.

- a) 120° Q2 b) 309° Q4 c) 17° Q1





Draw the rotation angle in standard position.



Angles with the same terminal arm are called *coterminal angles*.

Since 150° is the measure of the smallest positive rotation angle coterminal with the angles in Class Example #2, it is called the *principal angle*.

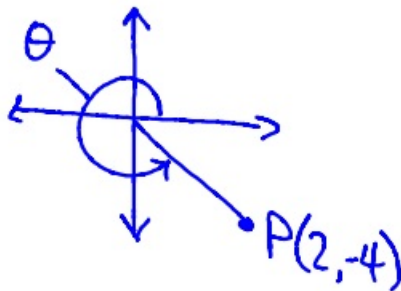
The principal angle will always have a measure between 0° and 360° .

There are infinitely many angles that are coterminal with a given angle.

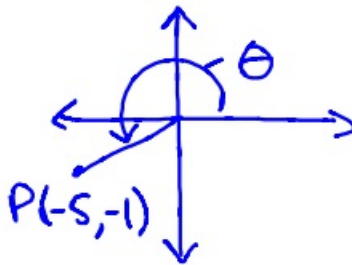


The point P lies on the terminal arm of the angle θ° . Draw the angle θ° in standard position.

a) $P(2, -4)$



b) $P(-5, -1)$



Complete Assignment Question #1 - #2

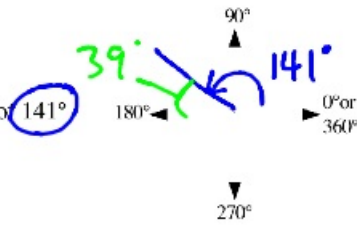
Reference Angles

$\leq 90^\circ$

A **reference angle** is the acute angle formed between the terminal arm of the rotation angle and the x-axis.

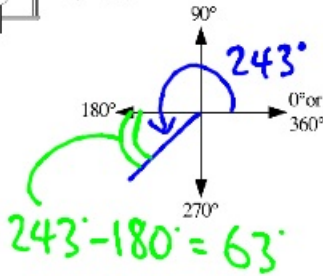
The diagram shows the terminal arm of a rotation angle of 141° with a reference angle of 39° .

Mark 141° and 39° on the diagram.

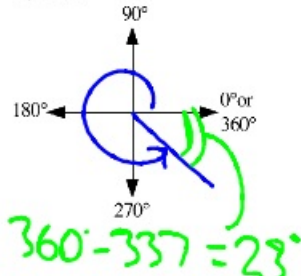


In each case, sketch the rotation angle and state the reference angle.

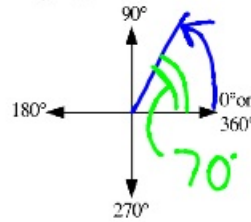
a) 243°



b) 337°



c) 70°



a) On the grid, draw a reference angle of 58° in each of quadrants one to four.

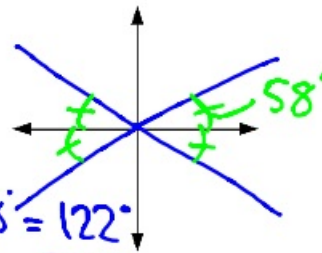
b) State the measure of the rotation angle in each quadrant.

In Q1, rotation $\angle = 58^\circ$

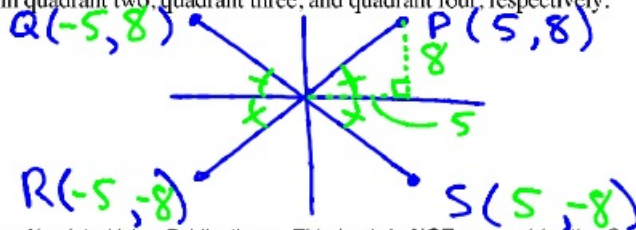
In Q2, rotation $\angle = 180^\circ - 58^\circ = 122^\circ$

In Q3, rotation $\angle = 180^\circ + 58^\circ = 238^\circ$

In Q4, rotation $\angle = 360^\circ - 58^\circ = 302^\circ$



c) Let $P(5, 8)$ be a point on the terminal arm of the rotation angle in quadrant one. State the coordinates of points Q , R , and S which are on the terminal arms of the rotation angles in quadrant two, quadrant three, and quadrant four, respectively.



Class Ex. #6



Determine the measure of the rotation angle, x , $0^\circ \leq x < 360^\circ$, given the reference angle and the quadrant.

Reference Angle	Quadrant	Sketch	Rotation Angle
25°	2		$180^\circ - 25^\circ = 155^\circ$
60°	4		$360^\circ - 60^\circ = 300^\circ$
8°	3		$180^\circ + 8^\circ = 188^\circ$
39°	1		39°
90°	between 3 and 4		$180^\circ + 90^\circ = 270^\circ$

Class Ex. #7



Determine three angles between 0° and 360° which have the same reference angle as a rotation angle of 256° .

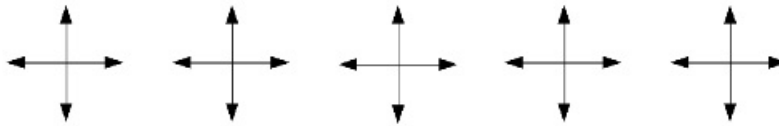
Complete Assignment Question #3 - #19

Do #1-19

Assignment

1. Sketch the following rotation angles in standard position, and state the quadrant in which the angle terminates.

- a) 135° b) 300° c) 190° d) 70° e) 270°



2. In each case, the given point is on the terminal arm of an angle of θ° . Draw the angle θ° in standard position.

- a) $P(7, -4)$ b) $Q(-2, 3)$ c) $R(-1, -4)$