# Trigonometry - Angles and Ratios Lesson #3:
## Applications of Reference Angles and the CAST Rule

### *Overview*

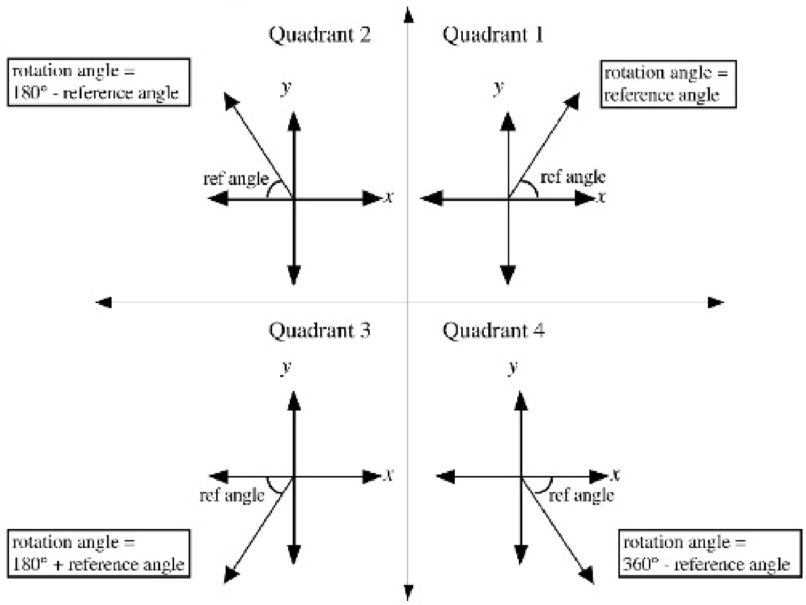
In this lesson, we use our knowledge of rotation and reference angles, and the CAST rule to:

**i)** determine the exact trigonometric ratios for rotation angles from 0° to 360° given a point on the terminal arm.

**ii)** determine trigonometric ratios for a rotation angle from 0° to 360° given a different trigonometric ratio for the angle.

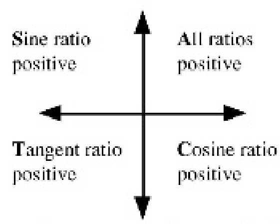
### *Review*

**The reference angle for any rotation angle is the acute angle between the terminal arm of the rotation angle and the *x*-axis.**




We can determine the sign of a trigonometric ratio in a particular quadrant:

- by the **CAST** rule or
- by remembering to "**A**dd **S**ugar **T**o **C**offee"




The trigonometric ratios for an angle in standard position with a point $P(x, y)$ on the terminal arm and $OP = r$ are

$$\sin\,\theta = \frac{y}{r} \qquad \cos\,\theta = \frac{x}{r} \qquad \tan\,\theta = \frac{y}{x}$$

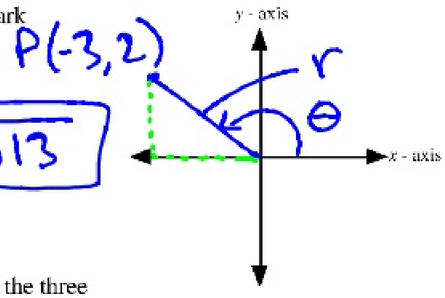
## *Exact Values of Trigonometric Ratios Given a Point on a Terminal Arm*

In the previous lesson, we were able to determine the exact values of the trigonometric ratios given a point on the terminal arm of a rotation angle in quadrant one.
In this lesson, we extend the method into quadrants two to four.

*Class Ex. #1*

The point $P(-3, 2)$ lies on the terminal arm of an angle $\theta$ in standard position. Complete the following procedure to determine the values of the primary trigonometric ratios.

**a)** Sketch the rotation angle on the grid and mark the point $P(-3, 2)$ on the terminal arm.




**b)** Calculate the exact length of $OP = r$.

$r=\sqrt{x^2+y^2}$

$r=\sqrt{(-3)^2+2^2}$

$r=\sqrt{9+4}$

$\boxed{r=\sqrt{13}}$

**c)** Use $x = -3$, $y = 2$ and $r$ from above to write the three trigonometric ratios for angle $\theta$.

$\sin\theta = \dfrac{y}{r} = \dfrac{2}{\sqrt{13}} \cdot \dfrac{\sqrt{13}}{\sqrt{13}} = \boxed{\dfrac{2\sqrt{13}}{13}}$

$\cos\theta = \dfrac{x}{r} = \dfrac{-3}{\sqrt{13}} \cdot \dfrac{\sqrt{13}}{\sqrt{13}} = \boxed{\dfrac{-3\sqrt{13}}{13}}$

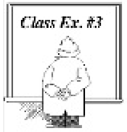
$\tan\theta = \dfrac{y}{x} = \boxed{\dfrac{2}{-3}}$

*Class Ex. #2*

The point $(-4, -2)$ lies on the terminal arm of an angle $\theta$ in standard position. Determine the exact value of $\sin\theta$.

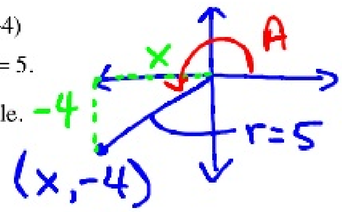
**Complete Assignment Questions #1 - #3**

## *Value of a Trigonometric Ratio Given a Different Trigonometric Ratio*




Angle $A$ terminates in the third quadrant with $\sin A = -\frac{4}{5}$. Complete the following procedure to determine the values of $\cos A$ and $\tan A$.

**a)** Since $\sin A = -\frac{4}{5} = \frac{y}{r}$, we know that the point $(x, -4)$ lies on the terminal arm in the third quadrant with $r = 5$. Sketch a diagram, draw the reference triangle and mark $x$, $y = -4$, and $r = 5$ on the reference triangle.




**b)** Use $x^2 + y^2 = r^2$ to determine the value of $x$.
*(Note that in quadrant three, the value of $x$ must be negative).*

$x^2 + (-4)^2 = 5^2$ → $x = \pm 3$

$x^2 = 25 - 16$

$x^2 = 9$

$\boxed{x = -3}$

**c)** Use the values of $x$, $y$, and $r$ to determine the exact values of $\cos A$ and $\tan A$.

$\cos A = \frac{x}{r} = \boxed{\frac{-3}{5}}$  $\tan A = \frac{y}{x} = \frac{-4}{-3} = \boxed{\frac{4}{3}}$

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If $\underbrace{\tan \theta = -\frac{2}{3} \text{ and } \cos \theta \text{ is positive}}$, then find the exact value of $\sin \theta$.

↳ in Q4 → x +ve, y -ve

S | A
T | C

$\tan \theta = \frac{y}{x} = -\frac{2}{3}$ → $x = 3$, $y = -2$

$r = \sqrt{3^2 + (-2)^2}$

$r = \sqrt{9 + 4}$

$r = \sqrt{13}$

$\sin \theta = \frac{y}{r} = \frac{-2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \boxed{\frac{-2\sqrt{13}}{13}}$

**Complete Assignment Questions #4 - #11**



Do #1-11 (not 3,4)