

Trigonometry - Angles and Ratios Lesson #4: Solving Simple Trigonometric Equations

Solving Trigonometric Equations with the Domain $0^\circ \leq \theta \leq 360^\circ$

We can use the concepts of reference angles and signs of the trigonometric ratio to solve equations of the form $\sin \theta = a$, $\cos \theta = a$, or $\tan \theta = a$, where $0^\circ \leq \theta \leq 360^\circ$.

Use the following procedure to solve an equation such as $\sin \theta = 0.5$, where $0^\circ \leq \theta \leq 360^\circ$.

Step 1: Determine the quadrant(s) the angle will be in by looking at the sign of the ratio.

Step 2: Determine the reference angle (always between 0° and 90°) and draw a rough sketch in the appropriate quadrant(s). To determine the reference angle, use

2nd sin or 2nd cos or 2nd tan

of the absolute value of the given quantity.

Step 3: Determine the rotation angle(s) using the reference angle and the quadrant(s).

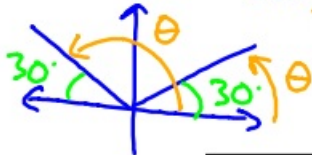


- Always check the given domain to determine which quadrants are valid in the calculation. Sometimes the domain is restricted to, for example, $0^\circ \leq \theta \leq 180^\circ$, or $90^\circ \leq \theta \leq 180^\circ$.

Class Ex. #1

Use the procedure above to solve $\sin \theta = 0.5$, where $0^\circ \leq \theta \leq 360^\circ$

sin +ve in Q1, Q2
ref $\angle = \sin^{-1}(0.5) = 30^\circ$



$\theta = 30^\circ$ in Q1
 $= 180 - 30 = 150^\circ$ in Q2

Class Ex. #2

Find the measure of x , to the nearest degree, where $0^\circ \leq x \leq 360^\circ$.

a) $\sin x = -0.8090$

b) $\cos x = -0.8090$

c) $\tan x = -2.4586$


sin -ve in Q3, Q4 | cos -ve in Q2, Q3 | tan -ve in Q2, Q4
 ref $\angle = \sin^{-1}(0.8090) = 54^\circ$ | ref $\angle = \cos^{-1}(0.8090) = 36^\circ$ | ref $\angle = \tan^{-1}(2.4586) = 68^\circ$
 $x = 180 + 54 = 234^\circ$ in Q3 | $x = 144^\circ$ in Q2 | $x = 112^\circ$ in Q2
 $= 360 - 54 = 306^\circ$ in Q4 | $= 216^\circ$ in Q3 | $= 292^\circ$ in Q4

Class Ex. #3

Solve the following equations if $0^\circ \leq \theta \leq 360^\circ$.

a) $\sin \theta = 1$

$$\begin{aligned} \text{sin +ve in Q1, Q2} \\ \text{ref } \angle = \sin^{-1}(1) \\ = 90^\circ \end{aligned}$$

$$\boxed{\theta = 90^\circ \text{ in Q1, Q2}}$$


b) $\cos \theta = 0$

$$\begin{aligned} \text{in all quadrants} \\ \text{ref } \angle = \cos^{-1}(0) \\ = 90^\circ \end{aligned}$$



$$\boxed{\theta = 90^\circ \text{ in Q1, Q2} \\ = 270^\circ \text{ in Q3, Q4}}$$

Class Ex. #4

Solve the equation $3 \tan \theta + 1 = 4$, $0^\circ \leq \theta \leq 180^\circ$

$$\frac{3 \tan \theta}{3} = \frac{3}{3}$$


$$\tan \theta = 1$$

tan +ve in Q1, ~~Q3~~

$$\begin{aligned} \text{ref } \angle = \tan^{-1}(1) \\ = 45^\circ \end{aligned}$$

$$\boxed{\theta = 45^\circ}$$

only consider
sol'n in Q1



Complete Assignment Questions #1 - #4

Solving Trigonometric Equations Outside the Domain $0^\circ \leq \theta \leq 360^\circ$

Use the following procedure to solve equations where the domain is outside of $0^\circ \leq \theta \leq 360^\circ$.

Step 1: Solve the equation with the domain $0^\circ \leq \theta \leq 360^\circ$ using the steps on the previous page.

Step 2: Using the concepts of coterminal angles, add or subtract 360° or multiples of 360° .



Solve the equation $\sqrt{3} \tan \theta + 1 = 0$, $0^\circ \leq \theta \leq 720^\circ$.

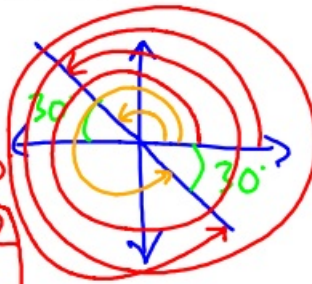
$$\frac{\sqrt{3} \tan \theta}{\sqrt{3}} = \frac{-1}{\sqrt{3}} \quad \text{ref } \angle = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = 30^\circ$$

$$\tan \theta = \frac{-1}{\sqrt{3}} \quad \text{tan-ve in Q2, Q4}$$

$$\theta = 150^\circ, 510^\circ \text{ in Q2}$$

$$= 330^\circ, 690^\circ \text{ in Q4}$$

$\swarrow 360^\circ + 150^\circ$
 $\nwarrow 360^\circ + 330^\circ$



To the nearest whole number, solve the equation $\cos x = -0.82$ where $-360^\circ \leq x \leq 0^\circ$.

Do # 1-7

Quiz L3+4 Friday

Complete Assignment Questions #5 - #9

Assignment

1. Solve the following equations, where $0^\circ \leq \theta \leq 360^\circ$.

a) $\cos \theta = \frac{1}{2}$

b) $\sin \theta = -\frac{\sqrt{3}}{2}$

c) $\tan \theta = -1$