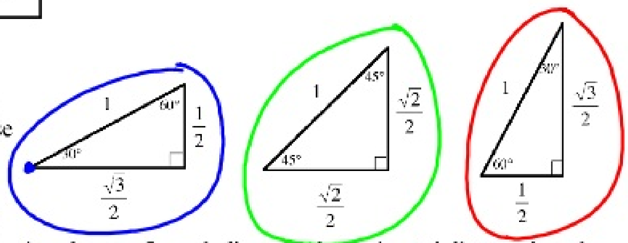
Trigonometry - Angle and Ratios Lesson #6: The Unit Circle

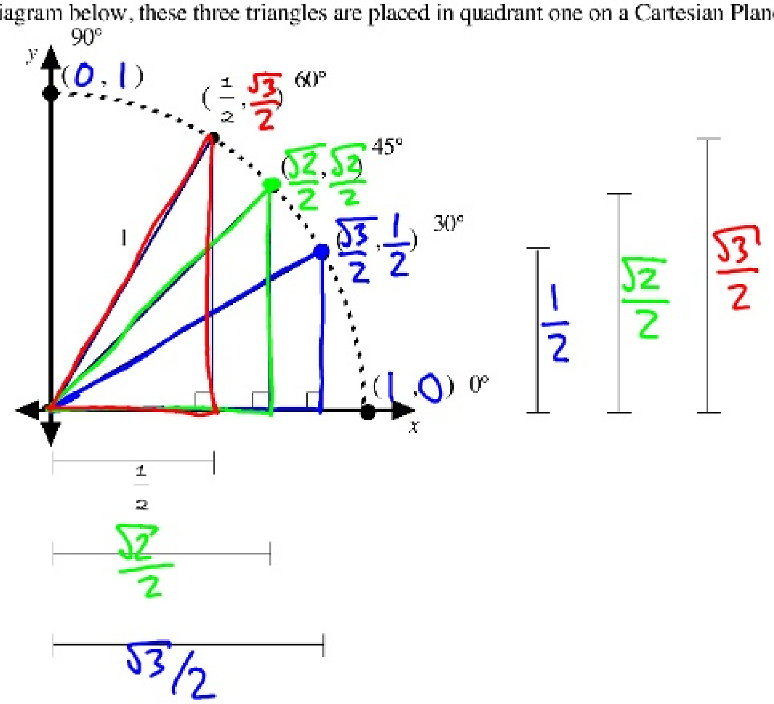
Creating The Unit Circle

An alternative method for determining exact values for trigonometric ratios of certain angles greater than 90° is to use the **unit circle.**



The illustration at the right shows the triangles developed from the investigation in the previous lesson. In each diagram, the horizontal distance is x, the vertical distance is y, and the hypotenuse is $r = 1$.

In the diagram below, these three triangles are placed in quadrant one on a Cartesian Plane.



a) Write the lengths of the horizontal and vertical line segments indicated.

b) Write the coordinates of the five points marked.

c) How do the coordinates of the points relate to the measure of the rotation angles?

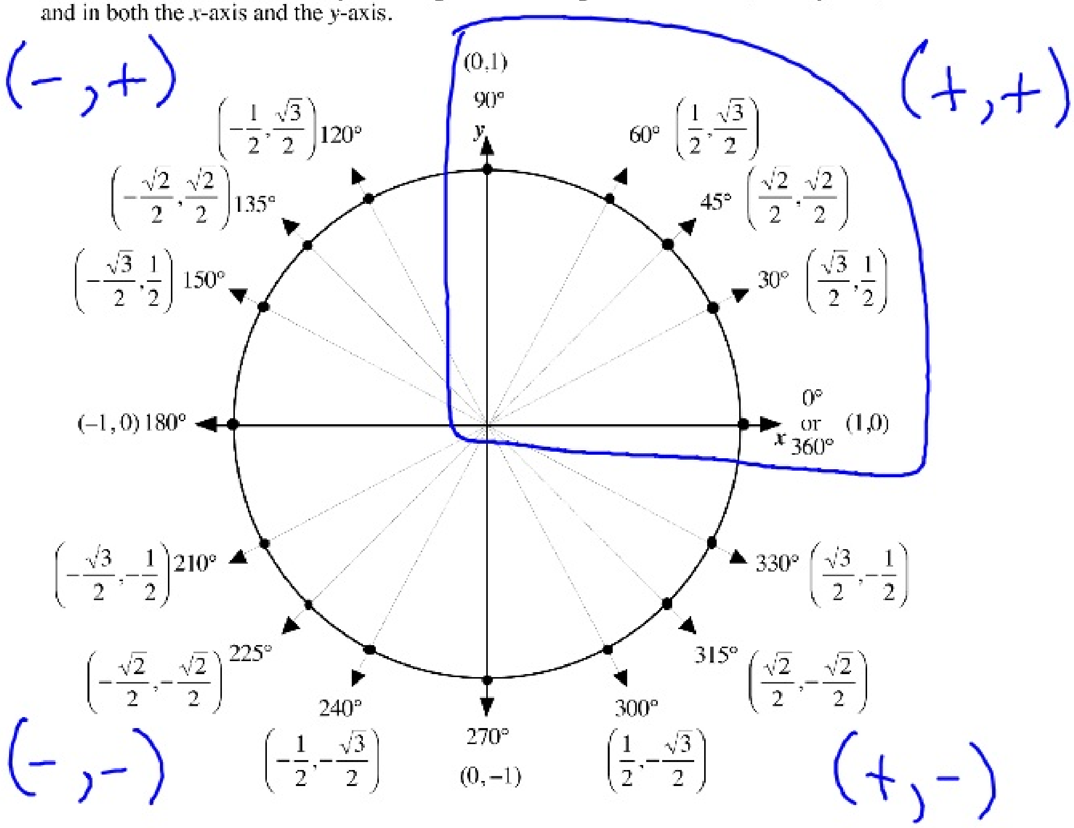
x coord = $\cos\theta$, y coord = $\sin\theta$

$(x, y) = (\cos\theta, \sin\theta)$

Complete Assignment Question #1

The Unit Circle

The unit circle can be formed by reflecting the above diagram in the x-axis, in the y-axis, and in both the x-axis and the y-axis.



The circle above, with a radius of one unit, is called the **unit circle** and it is important to understand how it works.

Recall the formulas $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$, and $\cot\theta = \frac{x}{y}$.

Note

- In the unit circle, where $r = 1$, we have

  $\sin\theta = y$ and $\cos\theta = x$

- Every point on the unit circle has coordinates (x, y) which can be written as $(\cos\theta, \sin\theta)$.

- $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{y}{x}$

Class Ex. #1

Use the unit circle to find the exact value of the trigonometric ratios for a rotation angle of 240°. Give each answer with a rational denominator.

sin 240° = $-\frac{\sqrt{3}}{2}$ cos 240° = $\frac{-1}{2}$ tan 240° = $\frac{\sin 240}{\cos 240} = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$

Class Ex. #2

Use the unit circle to find the exact value of

a) cos 135° **b)** tan 120° **c)** sin 180° **d)** tan 270°

a) $= \frac{-\sqrt{2}}{2}$

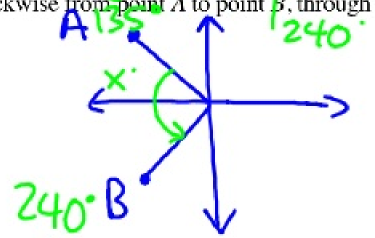
b) $= \frac{\sin 120^\circ}{\cos 120^\circ} = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$

c) $= 0$

d) $= \frac{\sin 270^\circ}{\cos 270^\circ} = \frac{-1}{0} =$ undefined.

Class Ex. #3

$A\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and $B\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ are two points on the unit circle. If an object rotates counterclockwise from point A to point B, through what angle has it rotated?



$x = 240 - 135$
$= 105°$

Class Ex. #4

Use a calculator to determine, to four decimal places, the coordinates of the point on the unit circle that corresponds to a rotation of 148°.

Do # 1-3, 8-12 (not 9)

Class Ex. #5

The point $T(-0.8829, 0.4695)$ lies on the unit circle. Determine the value of θ, where θ is the angle made by the positive x-axis and the line passing though T.

Note We now have two methods for determining exact values of trigonometric ratios of certain angles greater than 90°. Use either method.