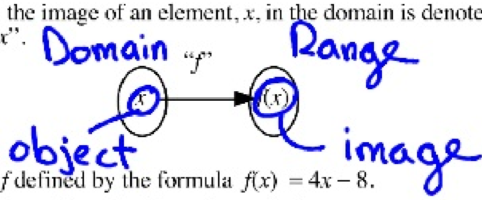
Quadratic Functions and Equations Lesson #1:
Connecting Zeros, Roots, and x-intercepts

Function and Function Notation

Recall the following information from previous math courses:

Function
A functional relation, or **function**, is a special type of relation in which each element of the domain is related to exactly one element of the range. If any element of the domain is related to more than one element of the range, then the relation is not a function.

Function Notation
Under a function, f, the image of an element, x, in the domain is denoted by $f(x)$, which is read "f of x".



Consider a function f defined by the formula $f(x) = 4x - 8$.
The notation $f(x) = 4x - 8$ is called **function notation**.

We can show that, under the function f, the image of 5 is 12. We write $f(5) = 12$.

function notation	equation of graph of function
$f(x) = 4x - 8$	$y = 4x - 8$
$f(5) = 4(5) - 8$	$y = 4(5) - 8$
$f(5) = 12$	$y = 12$

We can also show that, under the function f, the image of 2 is 0. We write $f(2) = 0$.

function notation	equation of graph of function
$f(x) = 4x - 8$	$y = 4x - 8$
$f(2) = 4(2) - 8$	$y = 4(2) - 8$
$f(2) = 0$	$y = 0$

We can say:
"The zero of the function $f(x) = 4x - 8$ is 2." "The root of the equation $y = 4x - 8$ is 2."

Zero(s) of a Function

A **zero of a function** is a value of the independent variable which makes the value of the function equal to zero. Zero(s) of a function can be found by solving the equation $f(x) = 0$.

Find the zero of the function f where $f(x) = 7x - 21$.

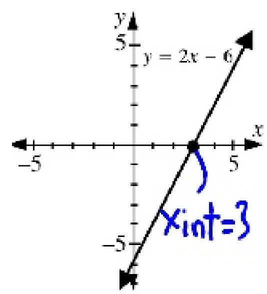
solve $0 = 7x - 21$
$21 = 7x$
$3 = x$

$\boxed{\text{The zero of } f \text{ is } 3}$

Investigation #1 *Connecting Roots, x-intercepts, and Zeros in a Linear Relation*

a) The graph of $y = 2x - 6$ is shown. Determine the x-intercept of the graph algebraically and graphically.

x-int → $y = 0$
solve $0 = 2x - 6$
$6 = 2x$ → $\boxed{3 = x\text{int}}$



b) Determine the root of the equation $2x - 6 = 0$.

$2x = 6$
$x = 3$

$\boxed{\text{Root is } 3}$

c) State the connection between the x-intercepts of the graph of $y = 2x - 6$ and the roots of the equation $2x - 6 = 0$.

Same number

d) Consider the function $f(x) = 2x - 6$. What is the zero of the function?

$0 = 2x - 6$
$6 = 2x$
$3 = x$

$\boxed{\text{Zero of } f \text{ is } 3}$

e) What is the connection between the x-intercepts of the graph of $y = 2x - 6$, the roots of the equation $2x - 6 = 0$, and the zero of the function $f(x) = 2x - 6$?

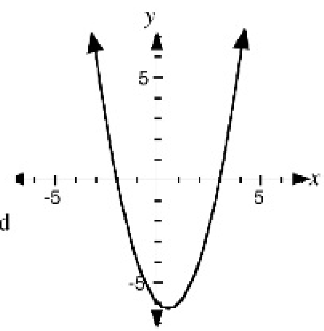
Same number

Investigation #2 *Connecting Roots, x-intercepts, and Zeros in a Quadratic Relation*

a) Jacques is determining the roots of the equation $x^2 - x - 6 = 0$.
He wrote the equation in factored form and used the Zero Product Law to determine the roots of the equation.

Complete his work to solve for x.

b) The graph of $y = x^2 - x - 6$ is shown.
State the x-intercepts of the graph and mark them on the grid.



c) i) State the connection between
the ***x*-intercepts** of the graph of $y = x^2 - x - 6$ and
the **roots** of the equation $x^2 - x - 6 = 0$.

ii) Explain the connection between
the **factors** of $x^2 - x - 6$ and
the **roots** of the equation $x^2 - x - 6 = 0$.

d) Consider the function $g(x) = x^2 - x - 6$. Determine the zeros of the function.

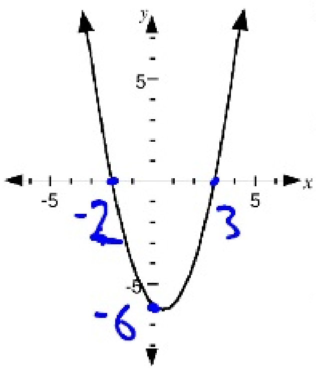
e) State the connection between
- the ***x*-intercepts** of the graph of $y = x^2 - x - 6$
- the **roots** of the equation $x^2 - x - 6 = 0$, and
- the **zeros** of the function $g(x) = x^2 - x - 6$

Class Ex. #2

a) Fill in the blanks in the following statement regarding the function with equation $y = f(x)$.

" The zeros of the function, the x-ints of the graph of the function, and the roots of the corresponding equation $y = 0$, are the same numbers."

b) The graph of $f(x) = x^2 - x - 6$ is shown. Fill in the blanks.



The *graph* of

$f(x)=x^2-x-6$

has ***x-intercepts***

$x =$ -2

and

$x =$ 3

with ***y-intercept***

$y =$ -6

The *function*

$f(x)=x^2-x-6$

$= (x+2)(x-3)$

has ***zeros***

-2 and 3

The *equation*

$x^2-x-6=0$

has ***roots*** $(x+2)(x-3)=0$

$x =$ -2

and

$x =$ 3

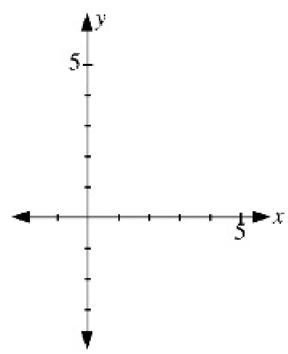
Class Ex. #3

Consider the equation $2x^2 - 7x + 3 = 0$.

a) Describe how the **zero feature** of a graphing calculator can be used to determine the roots of the equation.

b) Use a graphing calculator to determine the roots of the equation and sketch the graph on the grid provided.

c) Use the x-intercepts of the graph of $y = 2x^2 - 7x + 3$ to factor the expression $2x^2 - 7x + 3$.



d) What are the zeros of the function $f(x) = 2x^2 - 7x + 3$?

Note

Finding Zeros of a Function

To find the zeros of a function, $f(x)$, either

- substitute zero for $f(x)$ and find the roots of the resulting equation

  or
- graph the function and determine the x-intercepts of the graph

Finding the Roots of an Equation by Factoring

Finding the roots of a single variable equation may involve factoring. Except in the case of a linear equation, set the equation to zero before factoring.

Recall the following techniques for factoring common factors, difference of two squares, trinomials of the form $x^2 + bx + c = 0$, and trinomials of the form $ax^2 + bx + c = 0$.

Class Ex. #4

Find the roots of the following equations.

a) $x^2 + 8x = 33$

$x^2 + 8x - 33 = 0$

$(x-3)(x+11) = 0$

$\boxed{\text{Roots: } 3, -11}$

b) $6(4x + 5)(x - 3) = 0$

$\boxed{\text{Roots: } -\frac{5}{4}, 3}$

c) $2x^2 - 8 = 0$

$2(x^2 - 4) = 0$

$2(x+2)(x-2) = 0$

$\boxed{\text{Roots: } \pm 2}$

Class Ex. #5

For the following functions

i) find the zeros **ii)** find the y-intercept of the graph of the function

a) $f(x) = 5x^2 + 15x - 20$

i) $0 = 5(x^2 + 3x - 4)$

$0 = 5(x-1)(x+4)$

$\boxed{\text{zeros: } 1, -4}$

ii) $Yint = 5(0)^2 + 15(0) - 20$

$\boxed{Yint = -20}$

b) $f(x) = 3x^2 - 11x + 10$

i) $0 = 3x^2 - 6x - 5x + 10$

$0 = 3x(x-2) - 5(x-2)$

$0 = (x-2)(3x-5)$

$\boxed{\text{zeros: } 2, 5/3}$

ii) $Yint = 3(0)^2 - 11(0) + 10$

$\boxed{Yint = 10}$

c) $g(x) = 2x(2x + 1)$

i) $\boxed{\text{zeros: } 0, -1/2}$

ii) $Yint = 2(0)(2(0)+1)$

$\boxed{Yint = 0}$

Do #1-11 (not 5,7)