Quadratic Functions and Equations Lesson #10: Practice Test

1. Which one of the following statements is false?

   A. The zeros of the function $f(x) = x^2 + 4x - 5$ are the roots of the equation $x^2 + 4x - 5 = 0$.

   B. The x-intercepts of the graph of the function with equation $y = x^2 + 5x + 6$ are the factors of the expression $x^2 + 5x + 6$.

   C. If $x = 3$ is a root of the equation $f(x) = 0$, then $x - 3$ is a factor of $f(x)$.

   D. If $x + 7$ is a factor of the function with equation $y = f(x)$, then -7 is an x-intercept of the graph of the function.

2. The zeros of the function $f(x) = 3(x - 5)(3x + 2)$ are

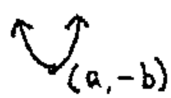
   A. $5, -\frac{2}{3}$ B. $-5, \frac{2}{3}$ C. $0, 5, -\frac{2}{3}$ D. $3, 5, -\frac{2}{3}$

   $x - 5 = 0 \Rightarrow x = 5$ $3x + 2 = 0 \Rightarrow x = -\frac{2}{3}$

3. The graph of the function $f(x) = (x - a)^2 - b$ is a parabola. Which of the following is a correct statement about the graph?

   A. There is a maximum point at $(-a, -b)$. B. There is a maximum point at $(a, -b)$.

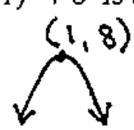
   C. There is a minimum point at $(-a, b)$. D. There is a minimum point at $(a, -b)$.



4. The range of the function $f(x) = -2(x - 1)^2 + 8$ is all real numbers such that

   A. $y \geq 8$ B. $y \leq 8$

   C. $y \geq 1$ D. $y \leq 1$



5. The coordinates of the vertex of the graph of the function $g(x) = x^2 - 4x + 11$ are

   A. (2, 15)

   B. (2, 7)

   C. (4, –5)

   D. (4, 11)

$$g(x) = x^2 - 4x + 11$$
$$= x^2 - 4x + 4 - 4 + 11$$
$$= (x - 2)^2 + 7$$

vertex $(2, 7)$

6. The graph of $y = x^2$ is translated 4 units left. The equation of the transformed graph is
A. $y = x^2 + 4$ B. $y = x^2 - 4$ C. $y = (x + 4)^2$ D. $y = (x - 4)^2$

7. The graph of $y = x^2$ undergoes two transformations to form the graph of $y = 4(x - 5)^2$. Which of the following is one of these transformations?

A. A horizontal translation 5 units left.
B. A vertical translation 4 units up.
C. A vertical stretch by a factor of 4 about the x-axis.
D. A vertical stretch by a factor of 5 about the x-axis.

Numerical Response 1. The graph of $y = (x - 5)^2 + b$ passes through the point $(3, 24)$. The value of b is _____.

(Record your answer in the numerical response box from left to right.)

2	0		

$24 = (3-5)^2 + b$

$24 = 4 + b$

$b = 20$

Numerical Response 2. The quadratic function $f(x) = x^2 - 12x + 41$ can be written in standard form $f(x) = (x - a)^2 + b$.

Write the value of a in the first box. Write the value of b in the second box.

(Record your answer in the numerical response box from left to right.)

6	5		

$x^2 - 12x + 41$

$= x^2 - 12x + 36 - 36 + 41$

$= (x - 6)^2 + 5$ $a = 6$ $b = 5$

8. The quadratic function $f(x) = x^2$ is transformed to $g(x) = -\frac{2}{3}(x + 3)^2 + 6$. The point $(6, 36)$ on the graph of f is transformed to which point on the graph of g?

A. $(3, -30)$ vertical stretch by a factor of $\frac{2}{3}$ about the x-axis
B. $(3, -28)$ reflection in the x-axis
C. $(3, -18)$ translation 3 units left and 6 units up
D. $(-6, 42)$ $(6, 36) \rightarrow (6, 24) \rightarrow (6, -24) \rightarrow (3, -18)$

9. The graph of a quadratic function has x-intercepts at m and $5m$, and a y-intercept of n. The equation of the axis of symmetry of the graph is

A. $x = \frac{m+n}{2}$ B. $x = \frac{6m+n}{3}$

C. $x = 3m$ D. $x = 6m$

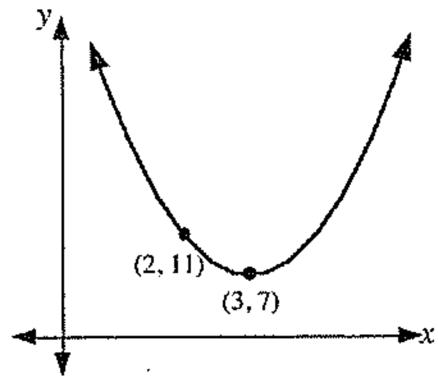
halfway between the x-intercepts

$\frac{m+5m}{2} = 3m$

$x = 3m$

Use the following information to answer question #10.

The graph of the quadratic function $f(x) = ax^2 + bx + c$ has vertex $(3, 7)$ and passes through the point $(2, 11)$.



10. The value of a is

A. -4

B. 4

C. -18

D. 18

$y = a(x-p)^2 + q$ vertex $(3,7)$

$y = a(x-3)^2 + 7$

$(2, 11) \rightarrow 11 = a(2-3)^2 + 7$

$11 = a + 7$

$a = 4$

11. The x-intercepts of the graph of the function $f(x) = x^2 - 8x - 4$ are

A. $2 \pm 4\sqrt{5}$

B. $4 \pm \sqrt{5}$

C. $4 \pm 2\sqrt{5}$

D. $-4 \pm 2\sqrt{5}$

$x^2 - 8x - 4 = 0$ $a = 1$ $b = -8$ $c = -4$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-4)}}{2(1)}$

$= \frac{8 \pm \sqrt{80}}{2} = \frac{8 \pm 4\sqrt{5}}{2}$

$= 4 \pm 2\sqrt{5}$

Numerical Response 3. Bailey used the quadratic formula to determine the positive root of the equation $2a^2 - 25a = 80$. To the nearest tenth, this root is ______ .

(Record your answer in the numerical response box from left to right.)

1	5	.	1

$2a^2 - 25a - 80 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{25 \pm \sqrt{(-25)^2 - 4(2)(-80)}}{2(2)} = \frac{25 \pm \sqrt{1265}}{4}$$

$a = 2 \quad b = -25 \quad c = -80$

$= -2.64\ldots,\ 15.14\ldots$

12. The roots of the equation $4x^2 + 4x - 5 = 0$ can be written in the form $x = \dfrac{-1 \pm \sqrt{A}}{2}$. The value of A is

$a = 4 \quad b = 4 \quad c = -5$

A. 6
B. 12
C. 24
D. 96

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{(4)^2 - 4(4)(-5)}}{2(4)} = \frac{-4 \pm \sqrt{96}}{8}$$

$$= \frac{-4 \pm 4\sqrt{6}}{8} = \frac{-1 \pm \sqrt{6}}{2} \qquad A = 6$$

Numerical Response 4. A student expresses the equation $y = -3x^2 + 12x - 11$ in the form $y = -a(x - b)^2 + c$.

Write the value of a in the first box.
Write the value of b in the second box.
Write the value of c in the second box.

(Record your answer in the numerical response box from left to right.)

3	2	1	

$y = -3x^2 + 12x - 11$

$y = -3(x^2 - 4x \quad) - 11$

$y = -3(x^2 - 4x + 4 - 4) - 11$

$y = -3(x - 2)^2 + 12 - 11$

$y = -3(x - 2)^2 + 1 \qquad a = 3 \quad b = 2 \quad c = 1$

13. When $y = 2x^2 - 7x + 6$ is converted to the form $y = a(x-p)^2 + q$, the value of q is

A. $-\frac{1}{8}$

B. $-\frac{37}{2}$

C. $\frac{47}{16}$

D. $\frac{97}{8}$

$y = 2x^2 - 7x + 6$

$y = 2\left(x^2 - \frac{7}{2}x\right) + 6$

$y = 2\left(x^2 - \frac{7}{2}x + \frac{49}{16} - \frac{49}{16}\right) + 6$

$y = 2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 6$

$y = 2\left(x - \frac{7}{4}\right)^2 - \frac{1}{8}$ $\quad q = -\frac{1}{8}$

14. The roots of the equation $2x^2 - 7x - 5 = 0$ are

A. $\frac{7 \pm \sqrt{89}}{2}$

B. $\frac{-7 \pm \sqrt{89}}{4}$

C. $\frac{7 \pm \sqrt{9}}{4}$

D. $\frac{7 \pm \sqrt{89}}{4}$

$a = 2 \quad b = -7 \quad c = -5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(-5)}}{2(2)}$

$= \frac{7 \pm \sqrt{89}}{4}$

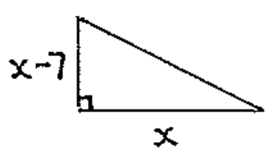
15. The shortest side of a right angled triangle is 7 cm less than the second shortest side. The sum of the squares of these two sides is equal to 289 cm^2. The perimeter of the triangle, in cm, is

A. 15

B. 23

C. 40

D. none of the above



Let x cm = length of second shortest side

$x - 7$ cm = length of shortest side

$x^2 + (x-7)^2 = 289$

$x^2 + x^2 - 14x + 49 = 289$

$2x^2 - 14x - 240 = 0$

$x^2 - 7x - 120 = 0$

$(x-15)(x+8) = 0$

$x = 15$ or $x = -8$ (reject since $x > 0$)

$x = 15$

$x - 7 = 8$

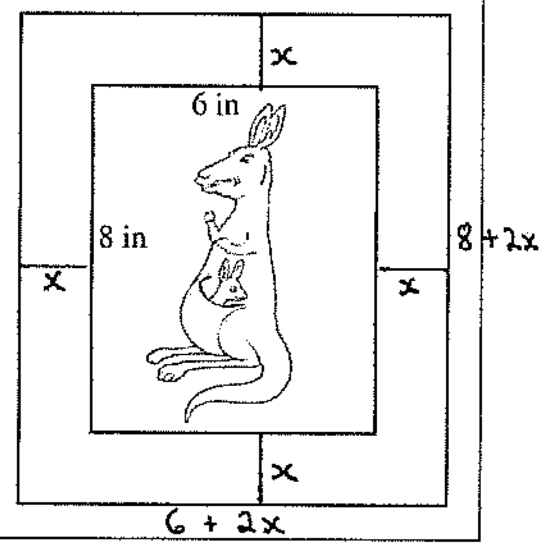
hypotenuse $= \sqrt{289} = 17$

perimeter $= 15 + 8 + 17 = 40$

Use the following information to answer the next question.

The diagram shows a photograph measuring 8 inches by 6 inches surrounded by a mat. The mat has the same width on all sides of the photograph.

The photograph and mat are put into a glass photo frame which just covers the outside of the mat.



Numerical Response

5. If the area of the glass surface is 120 square inches, determine the width of the mat to the nearest tenth of an inch.

(Record your answer in the numerical response box from left to right.)

2	.	0	

area $= (6+2x)(8+2x) = 120$

$48 + 28x + 4x^2 = 120$

$4x^2 + 28x - 72 = 0$

$4(x^2 + 7x - 18) = 0$

$4(x+9)(x-2) = 0$

$x = -9$ or $x = 2$

(reject since $x > 0$)

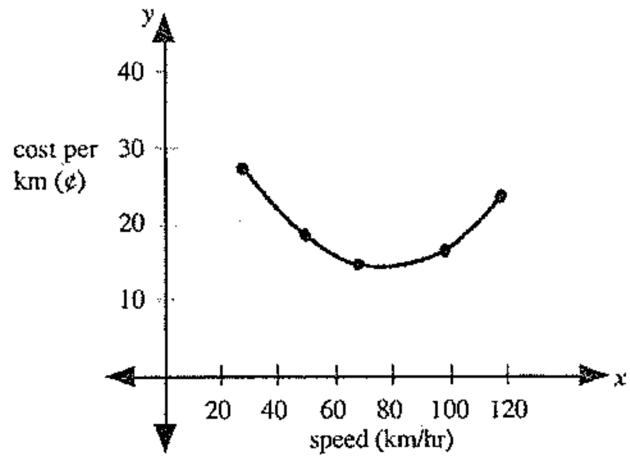
width $= 2.0$ in

Written Response - 5 marks

1. Kreshaun, a high school student, found that driving a truck can be a costly venture depending on how fast he drives. He knew from his Mechanics class that if he drives his truck too slowly, the cost per km is high because the engine does not run efficiently. He also knows from Physics class that if he drives his truck too fast, the cost per km is also high because of high wind resistance. He accumulated the following data.

Speed (km/hr)	30	50	65	100	120
Cost per kilometre	27.4¢	19.4¢	16.0¢	16.9¢	22.9¢

- If x represents the speed in km/h and y represents the cost per km in cents, plot the data on a Cartesian plane and join the points with a smooth curve.



- Looking at the graph, Kreshaun thought that the data could be modelled by a quadratic function with equation $y = ax^2 + bx + c$.
  He used the technique of quadratic regression to determine the equation $y = 0.005x^2 - 0.801x + 46.928$ as the best model for the data.

  Using the model above, determine the cost per km, to the nearest tenth of a cent, at a speed of 70 km/h.

  $x = 70$ $y = 15.358$ 15.4 cents per km

- Determine the speed, to the nearest km/h, if the cost is 20 cents per km.

  graph $y_2 = 20$ intersect at $x = 48$ and $x = 112.2$ speed is 48 km/h or 112 km/h.

- Which speed, to the nearest km/h, results in the lowest cost per kilometre? What is this cost to the nearest tenth of a cent?

  minimum point (80.1.., 14.847..) Speed = 80 km/h cost = 14.8 cents per km

- Does it make sense to extend the parabola to the left or right of the data points?

  No, because it is unlikely that the truck will travel at speeds less than 30 km/h or speeds greater than 120 km/h for an extended period of time.