Quadratic Functions and Equations Lesson #2:
Analyzing Quadratic Functions - Part One

Quadratic Function

A **quadratic function** is a function which can be written in the form

$$f(x) = ax^2 + bx + c, \text{ where } a, b, c \in R, \text{ and } a \neq 0$$

or in equation form as

$$y = ax^2 + bx + c, \text{ where } a, b, c \in R, \text{ and } a \neq 0$$

Quadratic Equation

A **quadratic equation** is an equation which can be written in the form

$$ax^2 + bx + c = 0, \text{ where } a, b, c \in R, \text{ and } a \neq 0.$$

The roots of the quadratic equation $ax^2 + bx + c = 0$ are
the zeros of the related quadratic function $f(x) = ax^2 + bx + c$.

General and Standard Forms

A quadratic function can be written in **general** or **standard** form.

General Form: $f(x) = ax^2 + bx + c$, **or** $y = ax^2 + bx + c$, where $a, b, c \in R$, and $a \neq 0$.

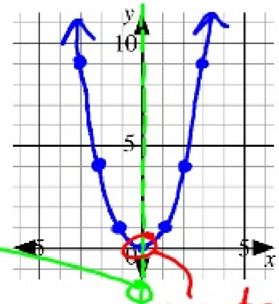
Standard Form: $f(x) = a(x - p)^2 + q$, **or** $y = a(x - p)^2 + q$, where $a, p, q \in R$, and $a \neq 0$.

In this unit we will study both the general form and standard form, beginning with the standard form in this lesson.

Analyzing the Graph of the Function with Equation $y = x^2$

- Graph the function with equation $y = x^2$ by completing the table of values. Join the points with a smooth curve. The graph of this function is called a parabola.

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



vertex (0,0)

- The **axis of symmetry** is the "mirror" line which splits the parabola in half. State the equation of the axis of symmetry for this parabola.

  $x = 0$

- The **vertex** of a parabola is where the axis of symmetry intersects the parabola. The vertex can represent a minimum point or maximum point depending on whether the parabola opens up or down.

  Label the vertex (V) on the graph and state its coordinates.

  (0,0)

- The maximum or minimum **value** of a quadratic function occurs at the vertex and is represented by the y-coordinate of the vertex. Complete the following:

  The minimum value of the function with equation $y = x^2$ is 0.

- State the domain and range of the function with equation $y = x^2$, $x \in R$.

  Domain: $\{x \mid x \in \mathbb{R}\}$ Range: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

Note: The following investigations can be completed as a class lesson or as an individual assignment. The process used in these explorations will be further developed in grade 12 mathematics.

Analyzing the Function with Equation $y = a(x - p)^2 + q,\ a = 1$

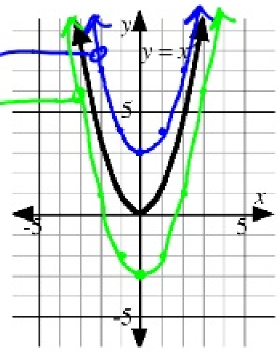
The next three investigations help us explore some general **transformations** on the graph of $y = x^2$ and the relationship they have to the standard form $y = a(x - p)^2 + q$, where $a = 1$.

A **transformation** is an operation which moves (or maps) a figure from an original position to a new position.

In each investigation, use a graphing calculator to sketch the equations.

Investigation #1

Analyzing the Graph of $y = x^2 + q$



The graph of $y = f(x) = x^2$ is shown.

a) Write an equation which represents each of the following:

- $y = f(x) + 3$ $y=x^2+3$
- $y = f(x) - 3$ $y=x^2-3$

b) Use a graphing calculator to sketch $y = f(x) + 3$ and $y = f(x) - 3$ on the grid.

c) Complete the following chart.

Function	*Equation Representing Function*	*Vertex*	*Max/Min Value*	*Equation of Axis of Symmetry*	*Description of Transformation*
$y = f(x)$	$y = x^2$	(0, 0)	min, 0	$x = 0$	no transformation
$y = f(x) + 3$	$y=x^2+3$	(0,3)	min,3	$x=0$	vertical translation 3 units up
$y = f(x) - 3$	$y=x^2-3$	(0,-3)	min,-3	$x=0$	vertical translation 3 units down
$y = f(x) + q$	$y=x^2+q$	(0,q)	min,q	$x=0$	vertical translation by q units

d) What is the effect of the **parameter**, q, on the graph of $y = x^2 + q$?

See e) below

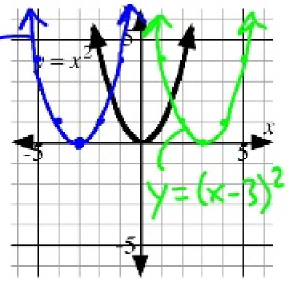
e) Compared to the graph of $y = x^2$, the graph of $y = x^2 + q$ results in

a vertical translation (or shift) of q units.

If $q > 0$, the parabola moves up. If $q < 0$, the parabola moves down.
+ve -ve

Investigation #2 **Analyzing the Graph of $y = (x - p)^2$**

$y=(x+3)^2$



The graph of $y = f(x) = x^2$ is shown.

a) Write an equation which represents each of the following:

- $y = f(x + 3)$ $\qquad$ • $y = f(x - 3)$

$y=(x+3)^2$ $\qquad$ $y=(x-3)^2$

b) Use a graphing calculator to sketch $y = f(x + 3)$ and $y = f(x - 3)$ on the grid.

c) Complete the following chart.

Function	*Equation Representing Function*	*Vertex*	*Max/Min Value*	*Equation of Axis of Symmetry*	*Description of Transformation*
$y = f(x)$	$y = x^2$	$(0, 0)$	min, 0	$x = 0$	no transformation
$y = f(x + 3)$	$y=(x+3)^2$	$(-3,0)$	min, 0	$x=-3$	horizontal translation 3 units left
$y = f(x - 3)$	$y=(x-3)^2$	$(3,0)$	min, 0	$x=3$	horizontal translation 3 units right
$y = f(x - p)$	$y=(x-p)^2$	$(p,0)$	min, 0	$x=p$	horizontal translation by p units

d) What is the effect of the **parameter**, p, on the graph of $y = (x - p)^2$?

See e) below

e) Compared to the graph of $y = x^2$, the graph of $y = (x - p)^2$ results in a <u>horizontal</u> translation (shift) of p units.

If $p > 0$, the parabola moves <u>right</u>. If $p < 0$, the parabola moves <u>left</u>.

+ve $\qquad$ -ve

Investigation #3 — Analyzing the Graph of $y = (x - p)^2 + q$

Consider the function $f(x) = x^2$.

a) Write an equation which represents $f(x + 2) - 4$.

b) Predict the transformations on $y = x^2$ in a). Use a graphing calculator to verify the results.

$y = (x+2)^2 - 4$

c) Complete the following chart.

Function	*Equation Representing Function*	*Vertex*	*Max/Min Value*	*Equation of Axis of Symmetry*	*Description of Transformation*
$y = f(x)$	$y = x^2$	$(0, 0)$	min, 0	$x = 0$	no transformation
$y = f(x + 2) - 4$	$y=(x+2)^2-4$	$(-2,-4)$	min, -4	$x=-2$	horizontal translation 2 left vertical translation 4 down
$y = f(x - p) + q$	$y=(x-p)^2+q$	(p,q)	min, q	$x=p$	horizontal translation p units vertical translation q units

Describe how the graphs of the following functions relate to the graph of $y = x^2$.

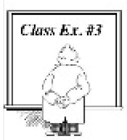
Do # 1-10

a) $y = (x + 10)^2$ **b)** $y = x^2 + 4$ **c)** $y + 8 = (x - 5)^2$

The following transformations are applied to the graph of $y = x^2$. Write the equation of the image function for each.

a) a horizontal translation of 5 units right

b) a translation of 6 units down and 4 units left



Write the coordinates of the image of the point $(3, 9)$ on the graph $y = x^2$ when a translation of two units up and seven units right is applied.

Complete Assignment Questions #1 - #10