Quadratic Functions and Equations Lesson #5:
Converting from General Form to Standard Form by Completing the Square

Review

- The **general form** of a quadratic function has the equation $y = ax^2 + bx + c$.
- The **standard form** of a quadratic function has the equation $y = a(x-p)^2 + q$.
- Writing a function in standard form enables us to analyze the function more easily e.g. we can determine the vertex, axis of symmetry and maximum / minimum value of the function.

Completing the Square

$(x+4)^2$ and $(x-5)^2$ are examples of **perfect squares.**

a) Expand the following perfect squares.

$(x+4)^2 = (x+4)(x+4) = x^2+8x+16$ $\qquad$ ~~$(x+7)^2 = (x+7)(x+7)$~~ = ____________

$(x-5)^2 = (x-5)(x-5) = x^2-10x+25$ $\qquad$ ~~$(x-1)^2 = (x-1)(x-1)$~~ = ____________

$(x+a)^2 = x^2+2ax+a^2$ $\qquad$ $(x-a)^2 = x^2-2ax+a^2$

b) Factor the following expressions into perfect squares.

$x^2+6x+9 = (x+3)^2$ $\qquad$ ~~$x^2+12x+36$~~ = ____________

$x^2-4x+4 = (x-2)^2$ $\qquad$ ~~$x^2-16x+64$~~ = ____________

c) Add an appropriate constant so that the following expressions can be written as perfect squares.

$x^2+2x+1 = (x+1)^2$ $\qquad$ ~~x^2+18x+~~ ___ = ________

$\hookrightarrow (1)^2$

$x^2-3x+9/4 = (x-3/2)^2$ $\qquad$ ~~$x^2-\frac{1}{4}x+$~~ ___ = ________

$\hookrightarrow (-3/2)^2$

The process of adding a constant term to a quadratic expression to make it a perfect square is called **completing the square.**

To complete the square of $x^2 + bx$, add $\left(\frac{1}{2}\text{coefficient of } x\right)^2$

i.e. add $\left(\frac{1}{2}b\right)^2$ to give $\left(x+\frac{1}{2}b\right)^2$.

Writing $f(x) = x^2 + bx + c$ in Standard Form by Completing the Square

Use the following process to convert a function of the form $f(x) = x^2 + bx + c$ into standard form.

Step 1: Take half of the numerical coefficient of the x-term and square it.

→ **Step 2**: Add and subtract the squared number in step 1. (This keeps the value of the function the same.)

→ **Step 3**: Form a perfect square, write it in factored form, and simplify.

Class Ex. #1

Express $y = x^2 + 10x + 16$ in completed square form. (i.e. standard form)
Use a graphing calculator to verify that both equations are represented by identical graphs.

$$y = \underbrace{x^2 + 10x + \underline{25}}_{} - \underline{25} + 16$$

$$(5)^2$$

$$\boxed{y = (x+5)^2 - 9}$$

Class Ex. #2

A function, f, is defined by $f(x) = x^2 - 9x - 20$.
Determine the minimum value of f by writing the function in standard form.

↳ y-coordinate of vertex.

$$f(x) = x^2 - 9x + \underline{81/4} - \underline{81/4} - 20$$

$$(-9/2)^2$$

$$f(x) = (x - 9/2)^2 - \frac{81}{4} - 20$$

$$\boxed{f(x) = (x - 9/2)^2 - \frac{161}{4}}$$

$$\boxed{\text{min value: } -\frac{161}{4}}$$

Complete Assignment Questions #1 - #4

Writing $f(x) = ax^2 + bx + c$ in Standard Form by Completing the Square

Step 1

Use the coefficient of x^2 as a common factor for the x^2 term and the x term only.

Step 2

Take half of the numerical coefficient of the x-term and square it.

Step 3

Add and subtract the squared number in step 2 inside the brackets.

Step 4

Multiply the term which has been subtracted in step 3 by the coefficient of x^2 and combine with the constant term.

Step 5

Form a perfect square within the brackets and write it in factored form.

Class Ex. #3

Convert $f(x) = 3x^2 - 18x + 20$ to standard form by completing the square. Determine whether the graph of the function f has a maximum or minimum value and state the value.

$f(x) = 3(x^2 - 6x + \underline{9} - \underline{9}) + 20$

$(-3)^2$

$f(x) = 3(x-3)^2 - 27 + 20$

$\boxed{f(x) = 3(x-3)^2 - 7}$ $\boxed{\text{min value} : -7}$

Class Ex. #4

Convert $y = 7 + 10x - 2x^2$ to standard form by completing the square. In what direction does the parabola open? What are the coordinates of the vertex of the parabola?

$y = -2x^2 + 10x + 7$

$y = -2(x^2 - 5x + \underline{25/4} - \underline{25/4}) + 7$

$(-5/2)^2$

Complete Assignment Questions #5 - #9

$y = -2(x - 5/2)^2 + \frac{25}{2} + 7$

$\boxed{y = -2(x-5/2)^2 + \frac{39}{2}}$

$\boxed{\text{opens down} \quad \text{vertex: } \left(\frac{5}{2}, \frac{39}{2}\right)}$

Do # 1-9