

Quadratic Functions and Equations Lesson #5: Converting from General Form to Standard Form by Completing the Square

Review

- The **general form** of a quadratic function has the equation $y = ax^2 + bx + c$.
- The **standard form** of a quadratic function has the equation $y = a(x - p)^2 + q$.
- Writing a function in standard form enables us to analyze the function more easily e.g. we can determine the vertex, axis of symmetry and maximum / minimum value of the function.

Completing the Square

$(x + 4)^2$ and $(x - 5)^2$ are examples of **perfect squares**.

a) Expand the following perfect squares.

$$(x + 4)^2 = (x + 4)(x + 4) = \underline{x^2 + 8x + 16} \quad \cancel{(x + 7)^2 = (x + 7)(x + 7) = \underline{\hspace{2cm}}}$$

$$(x - 5)^2 = (x - 5)(x - 5) = \underline{x^2 - 10x + 25} \quad \cancel{(x - 1)^2 = (x - 1)(x - 1) = \underline{\hspace{2cm}}}$$

$$(x + a)^2 = \underline{x^2 + 2ax + a^2} \quad (x - a)^2 = \underline{x^2 - 2ax + a^2}$$

b) Factor the following expressions into perfect squares.

$$x^2 + 6x + 9 = \underline{(x + 3)^2} \quad \cancel{x^2 + 12x + 36 = \underline{\hspace{2cm}}}$$

$$x^2 - 4x + 4 = \underline{(x - 2)^2} \quad \cancel{x^2 - 16x + 64 = \underline{\hspace{2cm}}}$$

c) Add an appropriate constant so that the following expressions can be written as perfect squares.

$$x^2 + 2x + \underline{\frac{1}{4}} = \underline{(x + 1)^2} \quad \cancel{x^2 + 18x + \underline{\hspace{1cm}} = \underline{\hspace{2cm}}}$$

$$x^2 - 3x + \underline{\frac{9}{4}} = \underline{(x - 3/2)^2} \quad \cancel{x^2 - \frac{1}{4}x + \underline{\hspace{1cm}} = \underline{\hspace{2cm}}}$$

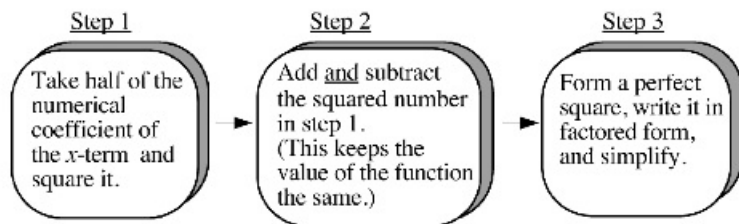
The process of adding a constant term to a quadratic expression to make it a perfect square is called **completing the square**.

To complete the square of $x^2 + bx$, add $\left(\frac{1}{2} \text{ coefficient of } x\right)^2$

i.e. add $\left(\frac{1}{2}b\right)^2$ to give $\left(x + \frac{1}{2}b\right)^2$.

Writing $f(x) = x^2 + bx + c$ in Standard Form by Completing the Square

Use the following process to convert a function of the form $f(x) = x^2 + bx + c$ into standard form.



Class Ex. #1



Express $y = x^2 + 10x + 16$ in completed square form. (i.e. standard form)
Use a graphing calculator to verify that both equations are represented by identical graphs.

$$y = x^2 + 10x + \frac{25}{1} - \frac{25}{1} + 16$$

↘ (5)² ↗

$$y = (x+5)^2 - 9$$

Class Ex. #2



A function, f , is defined by $f(x) = x^2 - 9x - 20$.
Determine the minimum value of f by writing the function in standard form.

↘ y-coordinate of vertex. ↗

$$f(x) = x^2 - 9x + \frac{81}{4} - \frac{81}{4} - 20$$

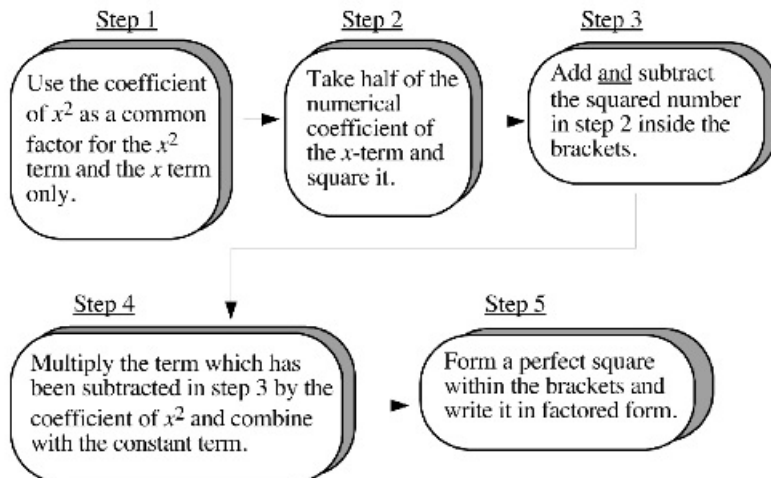
↘ (-9/2)² ↗

$$f(x) = \left(x - \frac{9}{2}\right)^2 - \frac{81}{4} - 20$$

$$f(x) = \left(x - \frac{9}{2}\right)^2 - \frac{161}{4}$$

min value: $-\frac{161}{4}$

Complete Assignment Questions #1 - #4

Writing $f(x) = ax^2 + bx + c$ in Standard Form by Completing the Square


Class Ex. #3

 Convert $f(x) = 3x^2 - 18x + 20$ to standard form by completing the square. Determine whether the graph of the function f has a maximum or minimum value and state the value.

$$f(x) = 3(x^2 - 6x + 9 - 9) + 20$$

$$f(x) = 3(x-3)^2 - 27 + 20$$

$$f(x) = 3(x-3)^2 - 7 \quad \text{min value: } -7$$

Class Ex. #4

 Convert $y = 7 + 10x - 2x^2$ to standard form by completing the square. In what direction does the parabola open? What are the coordinates of the vertex of the parabola?

$$y = -2x^2 + 10x + 7$$

$$y = -2(x^2 - 5x + \frac{25}{4} - \frac{25}{4}) + 7$$

Complete Assignment Questions #5 - #9

 Copyright © by Absolute Value Publications. This book is **NOT** covered by the Cancopy agreement.

$$y = -2(x - \frac{5}{2})^2 + \frac{25}{2} + 7$$

$$y = -2(x - \frac{5}{2})^2 + \frac{39}{2}$$

$$\text{opens down vertex: } (\frac{5}{2}, \frac{39}{2})$$

D₀ # 1-9