

Quadratic Functions and Equations Lesson #6: Solving Quadratic Equations - The Quadratic Formula

In previous lessons we have determined the roots of quadratic equations by graphing, and by factoring using inspection or decomposition.

In this lesson we will use

- the square root method
- the completing the square method
- the quadratic formula

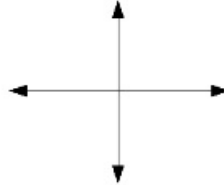
to solve quadratic equations.

Review

Class Ex. #1



Solve the equation $3x^2 + 13x - 10 = 0$ by graphing.
Use the sketch to illustrate your solution.



Class Ex. #2



Solve the following equations by factoring.

a) $x^2 + 7x - 18 = 0$

b) $3x^2 + 13x - 10 = 0$

The Square Root Method

The solution to the equation $x^2 = k$ is found by taking the square root of each side to get $x = \pm\sqrt{k}$.

Class Ex. #3



Use the square root method to solve the following quadratic equations.

a) $3x^2 = 27$

$$x^2 = 9$$

$$x = \pm\sqrt{9}$$

$$x = \pm 3$$

~~b) $(2x - 1)^2 = 64$~~

c) $(x + 3)^2 - 8 = 0$

$$(x + 3)^2 = 8$$

$$x + 3 = \pm\sqrt{8}$$

$$x = -3 \pm\sqrt{8}$$

$$x = -3 \pm 2\sqrt{2}$$

Completing the Square Method

This is an extension of the method used in Class Ex. #3c).

Class Ex. #4



a) Explain why the quadratic equation $3x^2 - 12x - 8 = 0$ cannot be solved by factoring.

$\frac{x}{-24} \pm \frac{-12}{-12}$ → there is no pair of numbers that has a product of -24 and a sum of -12

b) Solve the equation $3x^2 - 12x - 8 = 0$ by expressing the left side in completed square form and then using the square root method to complete solving the equation.

$$3(x^2 - 4x + \frac{4}{3} - \frac{4}{3}) - 8 = 0$$

$$3(x-2)^2 - 12 - 8 = 0$$

$$3(x-2)^2 - 20 = 0$$

$$3(x-2)^2 = 20$$

$$(x-2)^2 = \frac{20}{3}$$

$$x - 2 = \pm\sqrt{\frac{20}{3}}$$

$$x = 2 \pm \sqrt{\frac{20}{3}}$$

Complete Assignment Questions #1 - #4

Developing the Quadratic Formula

We will use the completing the square method to develop a formula that can be used to solve any quadratic equation of the form $ax^2 + bx + c = 0$.

Class Ex. #5

Solve the following equations by completing the square.

a) $2x^2 - 8x + 5 = 0$

b) $ax^2 + bx + c = 0$

The solution to Class Ex. #1b) is a formula which can be used to solve any quadratic equation of the form $ax^2 + bx + c = 0$. The formula is known as the **quadratic formula**.

Solving a quadratic equation by completing the square is rarely used as the quadratic formula is usually a more efficient method.

The Quadratic Formula

The quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has the roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Class Ex. 86



Find the roots of the following equations using the quadratic formula. Give answers as exact values in simplest form and to the nearest tenth.

a) $x^2 + 2x - 1 = 0$

$a=1, b=2, c=-1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{8}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{2}}{2} = \frac{-1 \pm \sqrt{2}}{1} = 0.4, -2.4$$

b) $4x^2 - 12x + 3 = 0$

$$4x^2 = 12x + 15$$

$$4x^2 - 12x - 15 = 0$$

$$a=4, b=-12, c=-15$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(-15)}}{2(4)}$$

$$x = \frac{12 \pm \sqrt{384}}{8}$$

$$x = \frac{12 \pm 8\sqrt{6}}{8} = \frac{3 \pm 2\sqrt{6}}{2} = 3.9, -0.9$$

Class Ex. 87



Find the zeros of the quadratic function $f(x) = -3x^2 + 4x + 1$.

Give answers as exact values in simplest form and to the nearest hundredth.

$$0 = -3x^2 + 4x + 1$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-3)(1)}}{2(-3)}$$

$$x = \frac{-4 \pm \sqrt{28}}{-6}$$

$$x = \frac{-4 \pm 2\sqrt{7}}{-6}$$

$$x = \frac{2 \pm \sqrt{7}}{3} = -0.22, 1.55$$

Complete Assignment Questions #5 - #13

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Do # 2, 5-12 (not 7)