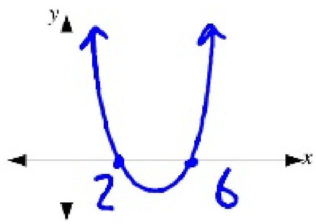
Quadratic Functions and Equations Lesson #7:
Roots of Quadratic Equations - The Discriminant

Review

Find the roots of the quadratic equation $x^2 - 8x + 12 = 0$ by each of the following methods.

i) by graphing



ii) by factoring

$(x-2)(x-6)=0$

$x=2,6$

iii) by completing the square

$x^2-8x+16-16+12=0$

$(x-4)^2-4=0$

$(x-4)^2=4$

$x-4=\pm\sqrt{4}$

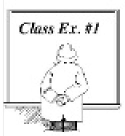
$x=4\pm 2=2,6$

iv) by the quadratic formula

$x=\dfrac{-(-8)\pm\sqrt{(-8)^2-4(1)(12)}}{2(1)}$

$x=\dfrac{8\pm\sqrt{64-48}}{2}$

$x=\dfrac{8\pm\sqrt{16}}{2}=\dfrac{8\pm 4}{2}=2,6$



Discuss when each of the following methods might be appropriate or not appropriate for solving a quadratic equation.

- by factoring using inspection or decomposition

- by quadratic formula

- by completing the square

- by graphing

Class Ex. #2

Form a quadratic equation and solve. $\frac{2}{a^2} + \frac{3}{a} = -1; \; a \neq 0$

$\frac{2 \cdot \cancel{a^2}}{\cancel{a^2}} + \frac{3 \cdot \cancel{a^2}}{\cancel{a}} = -1 \cdot a^2$

$2 + 3a = -a^2$

$a^2 + 3a + 2 = 0$

$(a+2)(a+1) = 0$

$\boxed{a = -2, -1}$

Complete Assignment Questions #1 - #3

Extension Enrichment

Investigating the Nature of the Roots of a Quadratic Equation

Insert the missing values.

Equation #1

$x^2 - 6x + 5 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

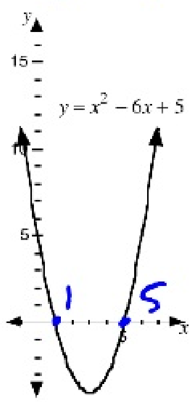
$x = \frac{6 \pm \sqrt{36 - 4(1)(5)}}{2}$

$= \frac{6 \pm \sqrt{16}}{2}$

$= \frac{6 + 4}{2}$ and $\frac{6 - 4}{2}$

$\therefore$ the roots are

$x = 5$ and $x = 1$



Equation #2

$x^2 - 6x + 9 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

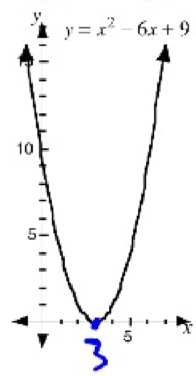
$x = \frac{6 \pm \sqrt{36 - 4(1)9}}{2}$

$= \frac{6 \pm \sqrt{0}}{2}$

$= \frac{6 + 0}{2}$ and $\frac{6 - 0}{2}$

$\therefore$ the roots are

$x = 3$ and $x = 3$



Equation #3

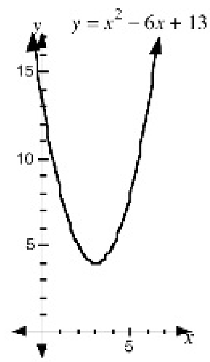
$x^2 - 6x + 13 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{6 \pm \sqrt{36 - 52}}{2}$

$= \frac{6 \pm \sqrt{-16}}{2}$

$\therefore$ the roots are not real



The Nature of the Roots of a Quadratic Equation

The roots of a quadratic equation are represented by the x-intercepts of the graph of the corresponding quadratic function.

The roots of a quadratic equation can be **equal or unequal** and **real or non-real**.

Consider the graphs from the previous page.

- In graph 1 the roots of the equation $x^2 - 6x + 5 = 0$ are real and unequal (distinct).
- In graph 2 the roots of the equation $x^2 - 6x + 9 = 0$ are real and equal.
- In graph 3 the roots of the equation $x^2 - 6x + 13 = 0$ are non-real.

The Discriminant

The nature of the roots of a quadratic equation can be determined without actually solving the equation or drawing its graph.

The number $b^2 - 4ac$, which appears under the radical symbol in the quadratic formula can be used to discriminate between the different types of roots, and is called the **discriminant**.

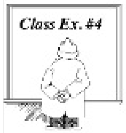
$$\boxed{\text{discriminant} = b^2 - 4ac}$$

a) Complete the table using the calculations from the investigation on the previous page.

Equation	Roots	Nature of Roots	$b^2 - 4ac$
$x^2 - 6x + 5 = 0$	1, 5	real, unequal	16 or +ve
$x^2 - 6x + 9 = 0$	3, 3	real, equal	0
$x^2 - 6x + 13 = 0$		non-real	-16 or -ve

b) Complete the following:

- If the discriminant $b^2 - 4ac = 0$, then the roots are real and equal.
- If the discriminant $b^2 - 4ac > 0$, then the roots are real and unequal
- If the discriminant $b^2 - 4ac < 0$, then the roots are non-real


Class Ex. #4

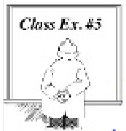
Determine the nature of the roots of the following equations without solving or graphing.

a) $6x^2 - x - 1 = 0$ **b)** $x^2 + 16 = 8x$ **c)** $5x^2 + 2x + 1 = 0$.

a)
b^2-4ac
$=(-1)^2-4(6)(-1)$
$=25$
real, unequal

b)
$x^2-8x+16=0$
b^2-4ac
$=(-8)^2-4(1)(16)$
$=0$
real, equal

c)
b^2-4ac
$=2^2-4(5)(1)$
$=-16$
non-real


Class Ex. #5

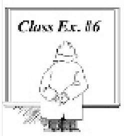
Determine for what value(s) of m the quadratic equation $x^2 - 8x + m$ has

a) real and distinct roots **b)** real and equal roots **c)** non-real roots

a)
$b^2-4ac>0$
$(-8)^2-4(1)m>0$
$64-4m>0$
$-4m>-64$
$\boxed{m<16}$

b)
$b^2-4ac=0$
$64-4m=0$
$-4m=-64$
$\boxed{m=16}$

c)
$b^2-4ac<0$
$64-4m<0$
$-4m<-64$
$\boxed{m>16}$


Class Ex. #6

a) State a condition for $b^2 - 4ac$ so that the equation $ax^2 + bx + c = 0$ has real roots.

$b^2-4ac \geq 0$

b) Given that the equation $ax^2 + bx + c = 0$ has real roots, state a condition for $b^2 - 4ac$ so that the roots are: **i)** rational, **ii)** irrational.

i) b^2-4ac is a perfect square

ii) b^2-4ac is not a perfect square

~~**c)** Show that the roots of the equation $(m-2)x^2 - (3m-2)x + 2m = 0$ are always real and rational.~~

Do #1-9

Quiz L5-7

Complete Assignment Questions #4 - #12